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FINITE ELEMENT ANALYSIS OF
OUT-OF-PLANE DISTORTION OF
WELDED PANEL STRUCTURES

Dong Baek Shin

FINITE ELEMENT ANALYSIS OF OUT-OF-PLANE
DISTORTION OF WELDED PANEL STRUCTURES

BY

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AND MARINE ENGINEERING

AT THE

MASSACHUSETTS INSTITUTE OF TECHNOLOGY



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DISTORTION OF WELDED PANEL STRUCTURES

by

DONG BAEK SHIN

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May 12, 1972, in partial fulfillment of the requirements for
the degrees of Ocean Engineer and Master of Science in Naval
Architecture and Marine Engineering.

ABSTRACT

Two dimensional distortions of a panel structure due to welding are analyzed by the finite element displacement method with an assumption of elastic deformation during welding process. Construction of stiffness matrices relevant to the welding problem is demonstrated for the one and the two dimensional cases. Computer programs are presented.

Computations have been carried out using the one dimensional experimental value of unconstrained angular change along the welded edge and its equivalent constrained welding moment as an input to the computer program.

These computed results are compared to the particular experimental values. The results indicate that an analytical approach using finite element method can predict the distortion phenomena of a complicated two dimensional structure with a reasonable degree of accuracy.

Several recommendations are made concerning further investigations aimed at evaluating relevant value of angular change and equivalent constrained welding moment.

Thesis Supervisor: Koichi Masubuchi

Title: Professor of Naval Architecture

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TABLE OF CONTENTS

	<u>Page</u>
TITLE PAGE	1
ABSTRACT	2
ACKNOWLEDGEMENT	4
TABLE OF CONTENTS	5
LIST OF FIGURES	7
LIST OF TABLES	8
NOMENCLATURE	9
I INTRODUCTION	11
A. Background	11
B. Previous Investigations	13
C. Aim and Purpose of Present Studies	18
II FORMULATION OF FINITE ELEMENT EQUATION	20
A. One Dimensional Case	20
B. Two Dimensional Case	31
1. Formulation of the Problem	31
2. Assembling Procedure	39
3. Symmetry and Boundary Conditions	39
4. Numerical Integration Method	41
5. Computer Programs	46
III RESULTS	50
IV DISCUSSION OF RESULTS	61

	<u>Page</u>
V CONCLUSIONS AND RECOMMENDATIONS	65
A. Conclusions	65
B. Recommendations	66
VI APPENDICES	
A. Definitions of Matrices	67
B. Example of Assembling	70
C. Description of Input Data	75
D. Listing of Programs	77
E. Sample Output	118
VII REFERENCES	123

LIST OF FIGURES

<u>Figure</u>		<u>Page</u>
1	Panel structure with longitudinal and transverse stiffeners	12
2	Distortion caused by angular change in two types of fillet welded structures	14
3	Variation of angular change of free fillet welds as a function of plate thickness and weight of electrode consumed	17
4	One-span beam element and the finite element local coordinate	21
5	Local coordinate of a two dimensional rectangular finite element and generalized displacement numbering	32
6	Various finite element having welded edge	38
7	Dimensions and element meshes of one-quarter of the plate for computer input	40
8	Ordinately and normalized coordinate for an element and 9-Gaussian points for numerical integration	43
9	Global coordinate system and shape of welded deflection	48
10	A panel structure analyzed by experiment and finite element method	49
11	Deflection comparison with experiment and finite element analysis	52
12	Angular changes along the welded boundary compared with experiment and finite element analysis	53
13	w_{max} at node 1 versus C with different θ_o	57
14	w_{max} at node 1 versus θ_o with different C	58
15	$(\partial w / \partial y)_{max}$ at node 6	59
16	$(\partial w / \partial x)_{max}$ at node 49	60

<u>Figure</u>		<u>Page</u>
17	w_{\max} at node 1 after superposition of figures 13 and 14	63
18	Elements and nodal points numbering in global and local coordinate for sample assembling	72

LIST OF TABLES

Table

1	Maximum deflection at node 1 with different combinations of C and θ_0	54
2	Maximum angular change $\partial w / \partial y$ at node 6 with different combinations of C and θ_0	55
3	Maximum angular change $\partial w / \partial x$ at node 49 with different combinations of C and θ_0	56
4	Input data layout form	76

NOMENCLATURE

a, b	Length of a finite element in x- and y-direction, respectively
C	Equivalent welding moment for one dimensional deflection
C_x	Equivalent welding moment along y-direction as in Figure 6
C_y	Equivalent welding moment along x-direction as in Figure 6
C	Coefficient matrix
D'	Matrix defined in equation (49)
\tilde{D}	Matrix defined as D'/β
E	Young's modulus of elasticity
$\tilde{E}_x, \tilde{E}_y, \tilde{G}, \tilde{H}, \tilde{q}$	Matrix defined in Appendix A
$\tilde{E}_x^T, \tilde{E}_y^T, \tilde{G}^T, \tilde{H}^T, \tilde{q}^T$	Transpose of the matrix $\tilde{E}_x, \tilde{E}_y, \tilde{G}, \tilde{H}$ and \tilde{q} , respectively.
I	$t^3/12$, in. ⁴ /in.
k	Stiffness matrix for one rectangular finite element
$\tilde{k}_{wx}, \tilde{k}_{wy}$	Additional stiffness matrix due to welding defined in equation (58b) and (58c), respectively
L_x, L_y	Length of one panel structure in x- and y-directions, respectively
M, N	Number of finite element in x- and y-directions, respectively
$P, \tilde{P}_x, \tilde{P}_y, \tilde{P}_{lx}, \tilde{P}_{ly}$	Matrix defined in equation (64), (65), (66) and (67), respectively
$\tilde{\Omega}_x, \tilde{\Omega}_y$	Load matrix due to welding defined in equations (58d) and (58e), respectively
t	Thickness of plate
T	Matrix defined in Appendix A

$\underline{\underline{T}}^{-1}$	Inverse matrix of $\underline{\underline{T}}$
U_T	Total strain energy
U_p	Elastic strain energy
U_w	Welding energy
w	Deflection in plate
w	Weight of electrode consumed, gram/cm.
w_i, w_j	Weight for numerical integration
x, y, z	Ordinarily coordinates for a finite element
x_i, y_j	Numerical integration points in normalized coordinates
β	Constant defined as $E t^3 / 12(1-\nu^2)$
ν	Poisson's ratio
θ_o	Free joint angular change
θ	Constrained angular change
δ	Variational notation

I INTRODUCTION

A. Background

Welding is used extensively in the fabrication of many structures, including ships, airplanes, buildings, pressure vessels, etc., providing many advantages over the techniques such as riveting, casting, and forging. However, distortion problems are always encountered during welding. Distortion often prevents the achievement of design tolerances, reduces joint strength by mismatching, and imparts initial deflection into structural members.

A typical structural member is a panel structure, which is composed of a flat plate and longitudinal and transverse stiffeners fillet welded to the plate, as shown in Figure 1. Angular changes produced along the fillet welds cause out-of-plane distortion of the panel. The excessive out-of-plane distortion reduces the buckling strength of the panel that is subjected to compressive loading.⁽³⁾ Corrugation failures of bottom plating in some welded cargo ships are believed to be due primarily to the reduction of buckling strength of the plating with excessive initial distortion.^(15,16,17)

Welding distortion is very complicated phenomena associated with many factors:

1. Heat conductivity of weldment which controls the distribution of temperature gradient of welded metal.
2. Thermal expansion coefficient which controls the expansion and shrinkage of welded metal.

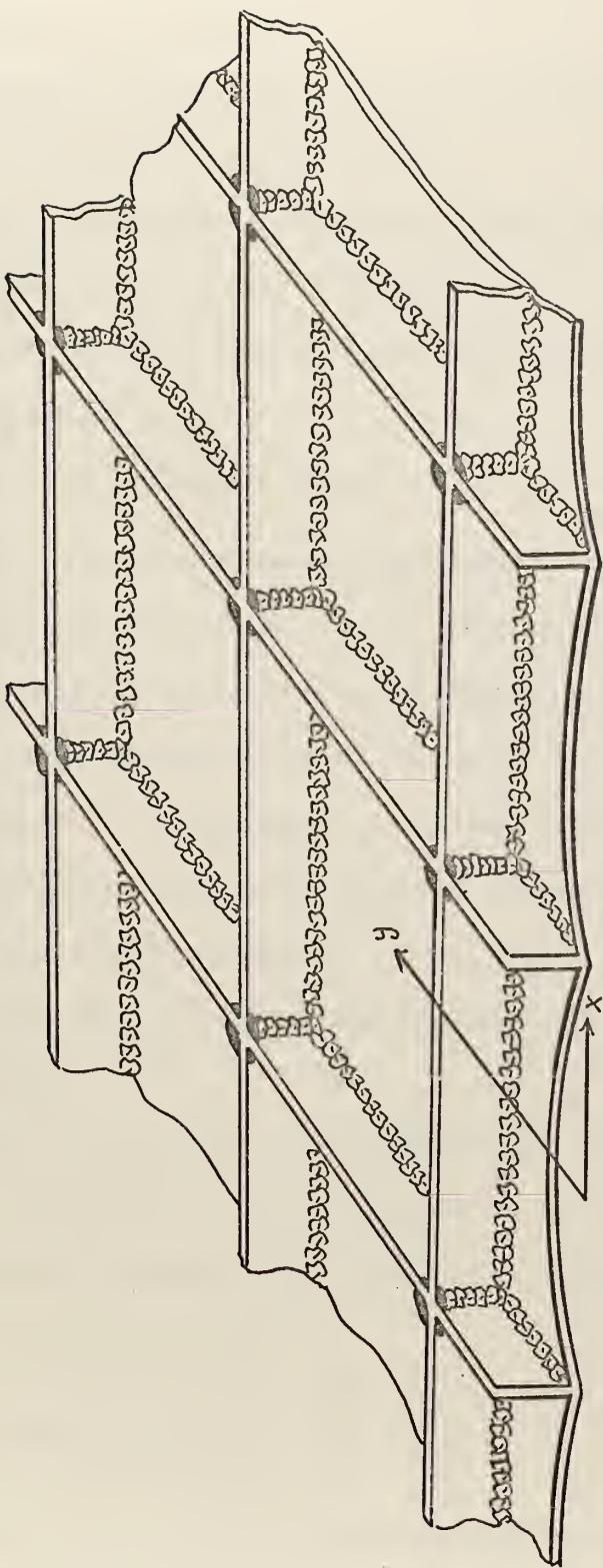


Figure 1 Panel Structure with Longitudinal and Transverse Stiffeners

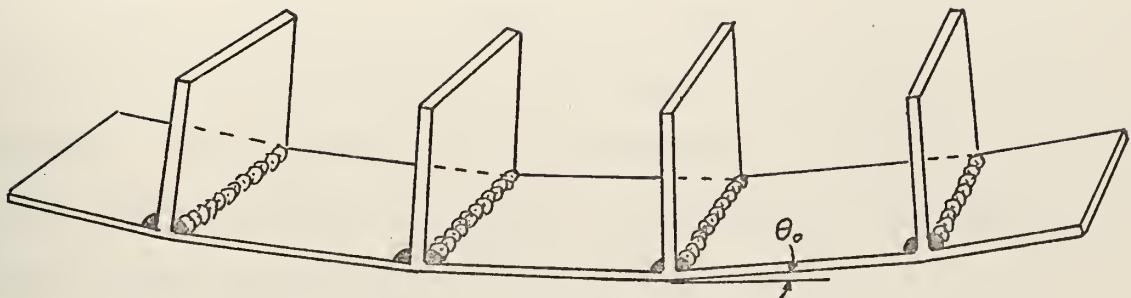
3. Moduli of elasticity which governs the rigidity against deformation.
4. Yielding strength which governs the size of the plastic zone near the welds.
5. Degree of constraint which governs the freedom of movement during weld deformation.
6. Amount of heat input used which governs the size of the melted zone.
7. Amount of weldment used.

Therefore, it is extremely difficult to analyze the welding deformation phenomena by a pure analytical way. Yet, only one dimensional deformation of panel structures has been analyzed with the assumptions of elastic deformation during welding process.⁽¹⁾ Obviously, welding deformations are neither pure elastic nor pure plastic phenomena, but the plastic zone near the weldments is so small compared to panel structural size that it is believed to be closer to the elastic behavior.

Therefore, the analytical approach of the one dimensional deformation prediction can be treated as an elastic deformation by considering moduli of elasticity, degree of constraint, and amount of weldment used.

B. Previous Investigations

Masubuchi, et. al.^(1,3) have investigated the one dimensional plate deformation during welding with a minimum strain energy concept using elastic bending theory.



a. Free Joint

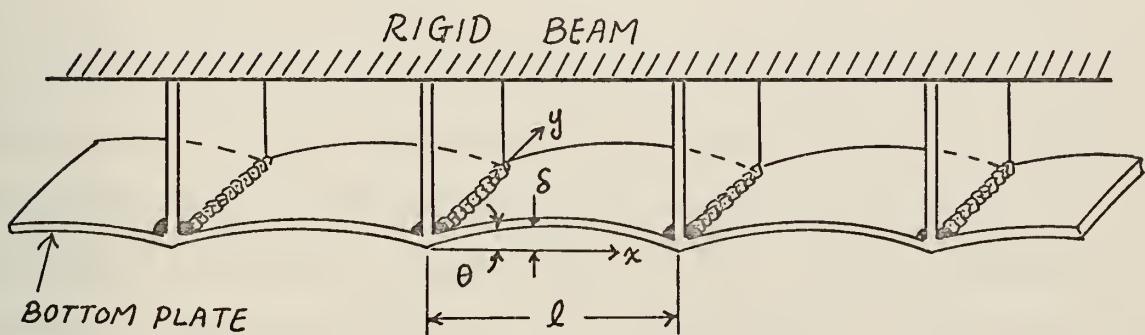
b. Constrained Joint
(framed structure)

Figure 2 Distortion Caused by Angular Change in Two Types of Fillet Welded Structures

If a fillet joint is free from external constraint, the joint simply bends to a polynomical form having a knuckle at the weld as shown in Figure 2a. However, if the joint is constrained by some means, a different type of distortion is produced. For example, if the stiffeners are welded to a rigid beam, as shown in Figure 2b, the angular changes at fillet welds cause wavy or arc-form distortion.

In the simplest case in which sizes of all welds are the same, the distortions of all spans are equal and the distortion, w , which can be expressed as follows:

$$\delta/l = [1/4 - (x/l - 1/2)^2] \cdot \theta \quad (1)$$

where δ = deflection of plate

θ_o = angular change at a fillet weld with constraint

l = length of span

The amount of angular change, θ , in constrained structures is smaller than that in a free joint, θ_o . This indicates that a certain amount of energy is necessary to decrease the angular change from θ_o to θ . If the necessary energy is represented by U_w , it may be given by the following equation.

$$U_w = \int_0^{\theta_o - \theta} \frac{dU_w}{d(\theta_o - \theta)} d(\theta_o - \theta) \quad (2)$$

On the other hand, the strain energy stored in the constrained plate, U_p , can be expressed using the elastic beam theory.

$$U_p = \frac{E'I}{l} \theta^2 \quad (3)$$

where $E' = E/l - v^2$

$$I = t^3/12$$

Since U_p increases but on the contrary U_w decreases as the constrained angle θ increases, the condition for equilibrium of this system requires that the total strain energy $U_t = U_w + U_p$ should be minimum. Furthermore, for the simplicity of the problem, the ratio of incremental welding energy change to angular change is assumed to be linear as in the following equation.

$$\frac{dU_w}{d(\theta_o - \theta)} = C(\theta_o - \theta) \quad (4)$$

where C = a coefficient determined by the weight of the deposited metal and by the welding procedures.

From equations (2) and (4), welding energy per unit length of width can be expressed in terms of C , θ , and θ_o ,

$$U_w = \frac{C}{2} (\theta_o - \theta)^2 \quad (5)$$

Accordingly, the condition of equilibrium is as follows:

$$\frac{\partial U_w}{\partial \theta} = C(\theta_o - \theta) + 2\frac{E'I}{l} = 0 \quad (6)$$

From equation (6), relation between θ and θ_o can be expressed,

$$\theta = \frac{\theta_o}{1 + \frac{2E'I}{l}/C} \quad (7)$$

The values of C can be determined by using relationship obtained by experimentally measuring θ_o and θ . Empirical formula for the calculation of C has been proposed by

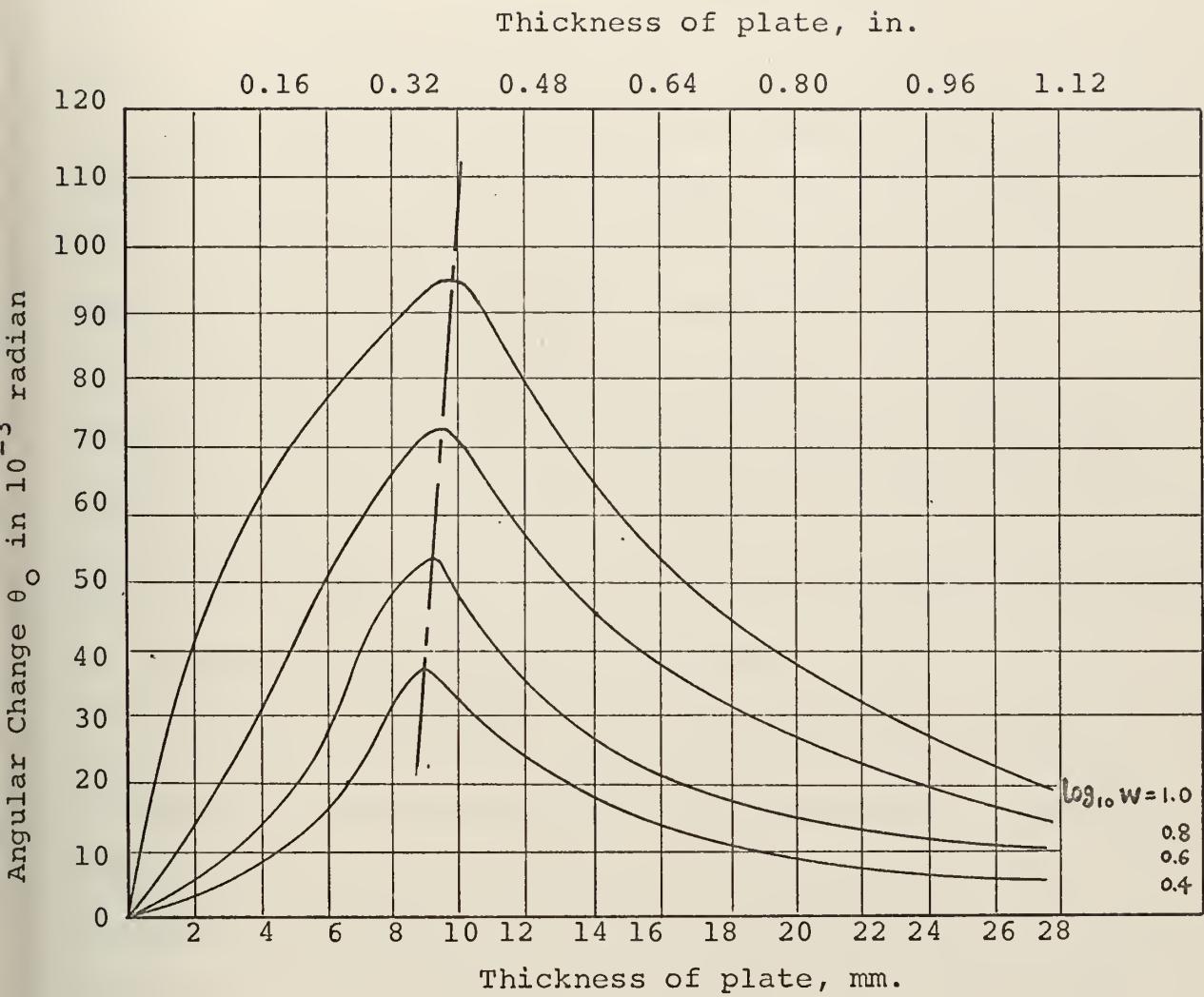


Figure 3 Variation of Angular Change of Free Fillet Welds, θ_0 , As a Function of Plate Thickness, t , and Weight of Electrode Consumed per Weld Length, W .

Masubuchi.⁽³⁾

$$C = t^4 / (l + W/5) \quad \text{kg-mm/mm radian} \quad (8)$$

where W = weight of electrode consumed, gram/cm.

t = thickness of plate, mm.

The experimental results of determining the unconstrained angular change as a function of plate thickness, t (mm), and weight of electrode consumed per weld length, W (gr/cm), have been given by Hirai, et. al.⁽²⁾ as in Figure 3.

Therefore, for a given plate geometry and welding condition, the deflection of the one dimensional case can easily be estimated by using equations (1), (7), (8), and Figure 3.

C. Aim and Purpose of Present Studies

As stated previously, welding deformation has been analyzed only for the one dimensional case, but in reality most of the structural members are encountered in two dimensional deformation, in which the deformation, w , varies along the x - and y -directions as well as the unconstrained angular change, θ , and equivalent constrained welding moment,

C. With the idea of previous investigations which state that the total strain energy in the plate during welding deformation becomes minimum, it is possible to extend the one dimensional deformation analysis to the two dimensional case. However, the simple mathematical solution using plate bending equation is rather difficult due to the uncertainty of the

equivalent load and the boundary conditions relevant to the welding deformation problem.

The difficulties can be easily overcome by formulating the problem into a finite element model because it can be self-adjusted to take care of the equivalent load and equivalent stiffness matrix due to welding. Furthermore, in this way the relation of deformation phenomena to the constrained angular change, θ , and the equivalent constrained welding moment C can be easily seen.

Therefore, it is intended to handle the two dimensional deformation with the finite element displacement technique.

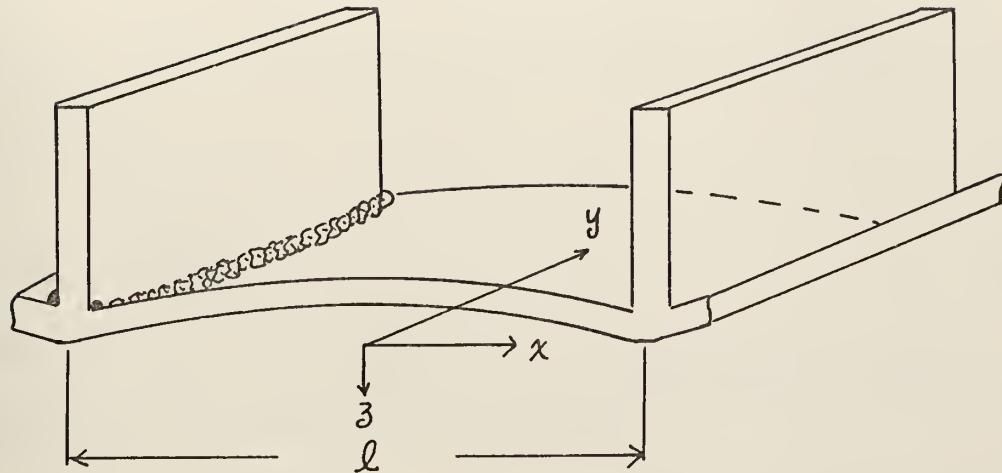
II FORMULATION OF FINITE ELEMENT EQUATION

A. One Dimensional Case

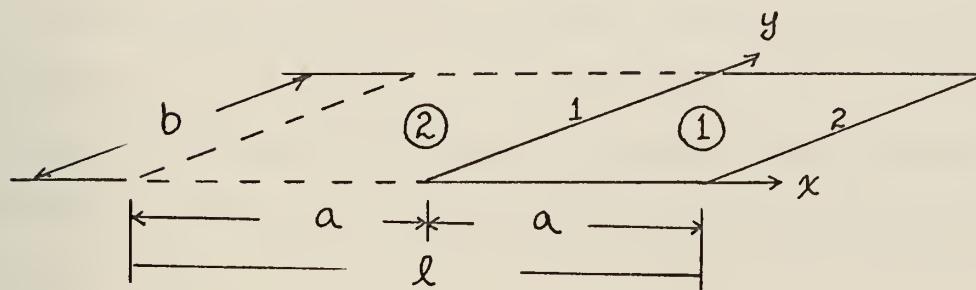
In this study, computer programs are programmed only for the two dimensional finite element analysis, but the one dimensional finite element formulations are presented to visualize the inside of the complex matrix assembling and computational procedure; furthermore, to compare the results to that of the previous investigators.

For the one dimensional case, as shown in Figure 2b, if deflections are elastic and small, the deflection can be calculated using elastic theory when the welding equivalent loads and the boundary conditions are prescribed. Even though the deformations are assumed to be elastic for the welded structural case, the loading and boundary conditions to be applied for this problem are not clear.

As introduced by Masubuchi, et. al.⁽¹⁾, if the concept of equivalent constrained welding moment, $C(\text{lb-in/in rad.})$ is being used, the uncertainty of loading conditions may be overcome; for the boundary condition it may be safe to assume to be simply supported for which only the angular change along the welded edge can be allowed. Furthermore, for simplicity of the problem, the deflections are considered only for the case of one span length of a simply supported beam to eliminate the complexity of interaction due to the statically indeterminate structural effects, as in Figure 4a.



a. Simplified one-span beam element



b. Local coordinate and nodal points for a finite beam element

Figure 4 One-span Beam Element and the Finite Element Local Coordinate

For the one dimensional plane stress, the strain energy can be expressed simply by:

$$U_p = 1/2 \iiint \epsilon_x \sigma_x dx dy dz \quad (9)$$

Using the stress-strain and strain-curvature relation for the beam element,

$$\sigma_x = E \epsilon_x \quad (10a)$$

$$\epsilon_x = -z \frac{d^2 w}{dx^2} \quad (10b)$$

Therefore, the strain energy can be expressed in terms of deflection and rigidity of the plate:

$$U_p = \frac{EI}{2} \int_0^a \left(\frac{d^2 w}{dx^2} \right)^2 dx \quad (11)$$

where

$$I = 2 \int_0^{t/2} \int_0^b z^2 dz dy = \frac{bt^3}{12}$$

On the other hand, the welding energy can be expressed in terms of θ_o , θ , and C , as in equation (5). For this problem it is more convenient to express the welding energy in terms of deflection rather than the angular changes. From equation (5),

$$U_w = \int_0^a \int_0^b \frac{C}{2} (\theta_o - \theta)^2 dy dx$$

the constrained angular change can be expressed by the equation

$$\theta = \partial w / \partial x \quad (12)$$

Therefore, the welding energy expressed,

$$U_w = \frac{bc}{2} \int_0^a \left(\theta_o - \frac{\partial w}{\partial x} \right)^2 dx \quad (13)$$

where the values of C and θ_o are constant which will be determined by the particular welding condition and the geometry.

The equilibrium condition for the system requires that the total strain energy has to be minimized, in other words, the variation of the total strain energy has to be zero, which becomes

$$\delta U_T = \delta U_p + \delta U_w = 0 \quad (14)$$

using equations (11) and (13), equation (14) becomes

$$\delta U_T = \delta \left[\frac{EI}{2} \int_0^a \left(\frac{d^2 w}{dx^2} \right)^2 dx \right] + \delta \left[\frac{bc}{2} \int_0^a \left(\theta_o - \frac{dw}{dx} \right)^2 dx \right] = 0$$

But the variation of the constant term is zero, therefore,

$$EI \int_0^a \left(\frac{d^2 \delta w}{dx^2} \right) \left(\frac{d^2 w}{dx^2} \right) dx + \frac{bc}{2} \int_0^a \left(\frac{d \delta w}{dx} \right) \left(\frac{dw}{dx} \right) dx - bc \int_0^a \left(\frac{d \delta w}{dx} \right) dx \quad (15)$$

For a beam element as shown in Figure 4b, the deflection can be assumed to be a general cubic function of x,

$$w = c_1 + c_2 x + c_3 x^2 + c_4 x^3 \quad (16)$$

and when the nodal displacements and the nodal angles are the unknown variables which are to be calculated, the coefficients (c_1, c_2, c_3, c_4) in equation (16) can exclusively be expressed in terms of the nodal displacements and angles considering the requirements of continuity at the nodal points

$$w(x=0) = w_1 = c_1$$

$$\frac{\partial w}{\partial x}(x=0) = w_{x_1} = c_2$$

$$w(x=a) = w_2 = c_1 + c_2 a + c_3 a^2 + c_4 a^3$$

$$\frac{\partial w}{\partial x}(x=a) = w_{x_2} = c_2 + 2c_3 a + 3c_4 a^2$$

The above requirements can be expressed in the matrix form,

$$\underline{q} = \underline{T} \underline{C} \quad (17)$$

where

$$\underline{T} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & a & a^2 & a^3 \\ 0 & 1 & 2a & 3a^2 \end{bmatrix} \quad (17a)$$

$$\underline{C} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} \quad (17b) \quad \text{and} \quad \underline{q} = \begin{bmatrix} w_1 \\ w_{x_1} \\ w_2 \\ w_{x_2} \end{bmatrix} \quad (17c)$$

From equation (17), \underline{C} can be expressed

$$\underline{C} = \underline{T}^{-1} \underline{q} \quad (18)$$

If the new matrix \underline{G} is defined such that

$$\underline{G} = \underline{T}^{-1} \quad (19)$$

then,

$$\underline{C} = \underline{G} \underline{q} \quad (20)$$

where

$$\underline{\underline{G}} = \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -3/a^2 & -2/a & 3/a^2 & -1/a \\ 2/a^3 & 1/a^2 & -2/a^3 & 1/a^2 \end{Bmatrix} \quad (21)$$

The second derivative of deflection function becomes,

$$\frac{d^2w}{dx^2} = 2c_3 + 6c_4x$$

which is in matrix form

$$\frac{d^2w}{dx^2} = \underline{\underline{H}} \underline{\underline{C}} \quad (22)$$

where $\underline{\underline{H}}$ is the matrix defined by

$$\underline{\underline{H}} = (0 \ 0 \ 2 \ 6x) \quad (23)$$

then,

$$\left(\frac{d^2\delta w}{dx^2} \right) \left(\frac{d^2w}{dx^2} \right) = \delta (\underline{\underline{H}} \underline{\underline{C}})^T (\underline{\underline{H}} \underline{\underline{C}}) = (\delta \underline{\underline{C}}^T) (\underline{\underline{H}}^T \underline{\underline{H}} \underline{\underline{C}})$$

but, using the equation (20),

$$\left(\frac{d^2\delta w}{dx^2} \right) \left(\frac{d^2w}{dx^2} \right) = (\delta \underline{\underline{q}}^T) (\underline{\underline{G}}^T \underline{\underline{H}}^T \underline{\underline{H}} \underline{\underline{G}} \underline{\underline{q}}) \quad (24)$$

The matrix expression for the first derivative of the deflection w can be expressed in the same manner,

$$\frac{dw}{dx} = c_2 + 2c_3x + 3c_4x^2$$

using $\underline{\underline{F}}$ and $\underline{\underline{C}}$

$$\frac{dw}{dx} = \underline{\underline{F}} \underline{\underline{C}} \quad (25)$$

where

$$\underline{\underline{F}} = \begin{pmatrix} 0 & 1 & 2x & 3x^2 \end{pmatrix} \quad (26)$$

Finally, using equation (20)

$$\frac{d\delta w}{dx} = (\delta \underline{\underline{q}}^T) (\underline{\underline{G}}^T \underline{\underline{F}}^T) \quad (27)$$

and

$$\left(\frac{d\delta w}{dx} \right) \left(\frac{dw}{dx} \right) = (\delta \underline{\underline{q}}^T) (\underline{\underline{G}}^T \underline{\underline{F}}^T \underline{\underline{F}} \underline{\underline{G}} \underline{\underline{q}}) \quad (28)$$

From equations (24), (27) and (28), the minimum energy equation (15) can be written in the matrix form,

$$\begin{aligned} & EI \int_0^a (\delta \underline{\underline{q}}^T) (\underline{\underline{G}}^T \underline{\underline{H}}^T \underline{\underline{H}} \underline{\underline{G}} \underline{\underline{q}}) dx \\ & + bc \int_0^a (\delta \underline{\underline{q}}^T) (\underline{\underline{G}}^T \underline{\underline{F}}^T \underline{\underline{F}} \underline{\underline{G}} \underline{\underline{q}}) dx \\ & - bc \int_0^a (\delta \underline{\underline{q}}^T) (\underline{\underline{G}}^T \underline{\underline{F}}^T) dx = 0 \end{aligned} \quad (29)$$

Noticing that the matrix $\underline{\underline{q}}$, $\underline{\underline{q}}^T$, $\underline{\underline{G}}$, and $\underline{\underline{G}}^T$ is not the function of x , but only of the function of dimension of the plate, and displacement and angle at the nodal points; therefore, equation (29) can be simplified to

$$\begin{aligned} & \delta \underline{\underline{q}}^T \{ EI \int_0^a (\underline{\underline{H}}^T \underline{\underline{H}}) dx \} \underline{\underline{G}} \underline{\underline{q}} \\ & + bc \int_0^a (\underline{\underline{G}}^T \underline{\underline{F}}^T \underline{\underline{F}}) dx \underline{\underline{G}} \underline{\underline{q}} \\ & - bc \int_0^a (\underline{\underline{G}}^T \underline{\underline{F}}^T) dx \} = 0 \end{aligned} \quad (30)$$

in which the only integrations necessary to be carried out are the bracketed quantities, which become after integration,

$$\int_0^a (\underline{H}^T \underline{H}) dx = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 4a & 6a^2 \\ 0 & 0 & 6a^2 & 12a^3 \end{pmatrix} \quad (31)$$

$$\int_0^a (\underline{F}^T \underline{F}) dx = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 2a & 3a^2 \\ 0 & 2a & 4a^2 & 6a^3 \\ 0 & 3a^2 & 6a^3 & 9a^4 \end{pmatrix} \quad (32)$$

$$\int_0^a (\underline{F}^T) dx = \begin{pmatrix} 0 \\ 1 \\ 2a \\ 3a^2 \end{pmatrix} \quad (33)$$

Equation (30) is the final equation for one element formulated by the finite element displacement method. The first quantity is defined as a stiffness matrix for an element,⁽¹¹⁾ the second is the additional stiffness matrix due to welding, and the third is the equivalent load due to welding. They are as follows:

Stiffness matrix for an element,

$$\underline{k} = EI \int_0^a [H^T H] dx G$$

$$= EI \begin{pmatrix} 12/a^3 & 6/a^2 & -12/a^3 & 6/a^2 \\ 6/a^2 & 4/a & -6/a^2 & 2/a \\ -12/a^3 & -6/a^2 & 12/a^3 & -6/a^2 \\ 6/a^2 & 2/a & -6/a^2 & 4/a \end{pmatrix} \quad (34)$$

Additional stiffness matrix due to welding,

$$\underline{k}_w = bc \int_0^a [F^T F] dx G$$

$$= \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & bc \end{pmatrix} \quad (35)$$

Load matrix due to welding,

$$Q = bc \int_0^a F^T dx$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \\ bc\theta_0 \end{pmatrix} \quad (36)$$

At this stage, assembling of equation (30) has to be considered when the one-span beam is divided by many finite elements, but the one-span beam element of Figure 4a is chosen to be two finite elements as in Figure 4b; then, only element 1 can be used to predict the deflection and angular

change at the nodal points by symmetric argument. In this case, the simultaneous equations to be solved are (from equation (30)),

$$\delta w_1 \left[\frac{6EI}{a^2} \left(\frac{2}{a} w_1 + w_{x_1} - \frac{2}{a} w_2 + w_{x_2} \right) \right] = 0$$

$$\delta w_{x_1} \left[\frac{2EI}{a} \left(\frac{3}{a} w_1 + 2w_{x_1} - \frac{3}{a} w_2 + w_{x_2} \right) \right] = 0$$

$$\delta w_2 \left[\frac{6EI}{a^2} \left(-\frac{2}{a} w_1 - w_{x_1} + \frac{2}{a} w_2 - w_{x_2} \right) \right] = 0$$

$$\delta w_{x_2} \left[\frac{2EI}{a} \left(\frac{3}{a} w_1 + w_{x_1} - \frac{3}{a} w_2 + \left(2 + \frac{abc}{2EI} \right) w_{x_2} - bc\theta_o \right) \right] = 0$$

(37a-d)

From the simply supported boundary condition at node 2, $w_2 = 0$; therefore, $\delta w_2 = 0$.

From symmetry condition at node 1, $w_{x_1} = 0$; therefore, $\delta w_{x_1} = 0$.

Furthermore, the displacement at node 1 and the angle at node 2 is not known, which means that the variations of these variables are not zero; therefore, the only way to satisfy equation (37a) and (37d) is the bracketed value in each equation has to be zero, which leads to final equation for w and w_{x_2} ,

$$\frac{2}{a} w_1 + w_{x_2} = 0$$

$$\frac{3}{a} w_1 + \left(2 + \frac{abc}{2EI} \right) w_{x_2} = 0 \quad (38a-b)$$

From equation (38a-b), displacement at the middle point and angular change at the edge expressed as,

$$w_1 = \frac{-a\theta_0}{2(1 + EI/abc)} \quad (39)$$

$$w_{x_2} = \frac{\theta_0}{2(1 + EI/abc)} \quad (40)$$

It is worth to compare the results of equations (6) and (7) with equations (39) and (40). In equations (39) and (40),

$$\frac{EI}{abc} = \frac{bt^3/12}{abc} = \frac{t^3/12}{ac}$$

and $a = l/2$ is to be used. Then,

$$w_1 = \frac{-l\theta_0}{4(1 + 2EI/lc)} \quad (41)$$

$$w_{x_2} = \frac{\theta_0}{1 + 2EI/lc} \quad (42)$$

where I is defined by unit width basis,^(1,6) which is $I = t^3/12$.

Equation (42) agrees with equation (7), as also does equation (41), if $x = l/2$ and θ of equation (7) is plugged into equation (1). The negative sign of equation (41) means that the coordinate and the positive angle at the edge has to be chosen, as in Figure 4a.

As shown, the results from the finite element method is exactly the same expression proposed by Masubuchi, et.al.^(1,3) for the one dimensional deformation, and in this way, the equivalent loads and the stiffness matrix due to welding become easily visualized.

B. Two Dimensional Case

1. Formulation of the Problem

In the two dimensional analysis, all procedures are the same as that of the one dimensional analysis except that the integrations are carried out by the numerical methods and the methods of assembling are mentioned. The deflection may be approximated by the bi-cubic function for one element, (12,14) as in Figure 5.

$$\begin{aligned} w = & c_1 + c_2x + c_3y + c_4xy + c_5x^2 + c_6y^2 + c_7x^3 \\ & + c_8x^2y + c_9xy^2 + c_{10}y^3 + c_{11}x^3y + c_{12}x^2y^2 \\ & + c_{13}xy^3 + c_{14}x^3y^2 + c_{15}x^2y^3 + c_{16}x^3y^3 \end{aligned} \quad (43a)$$

$$\begin{aligned} \frac{\partial w}{\partial x} = & c_2 + c_4y + 2c_5x + 3c_7x^2 + 2c_8xy + c_9y^2 \\ & + 3c_{11}x^2y + 2c_{12}xy^2 + c_{13}y^3 + 3c_{14}x^2y^2 \\ & + 2c_{15}xy^3 + 3c_{16}x^2y^3 \end{aligned} \quad (43b)$$

$$\begin{aligned} \frac{\partial w}{\partial y} = & c_3 + c_4x + 2c_6y + c_8x^2 + 2c_9xy + 3c_{10}y^2 \\ & + c_{11}x^3 + 2c_{12}x^2y + 3c_{12}xy^2 + 2c_{14}x^3y \\ & + 3c_{15}x^2y^2 + 3c_{16}x^3y^2 \end{aligned} \quad (43c)$$

$$\begin{aligned} \frac{\partial^2 w}{\partial x \partial y} = & c_4 + 2c_8x + 2c_9y + 3c_{11}x^2 + 4c_{12}xy \\ & + 3c_{13}y^2 + 6c_{14}x^2y + 6c_{15}xy^2 + 9c_{16}x^2y^2 \end{aligned} \quad (43d)$$

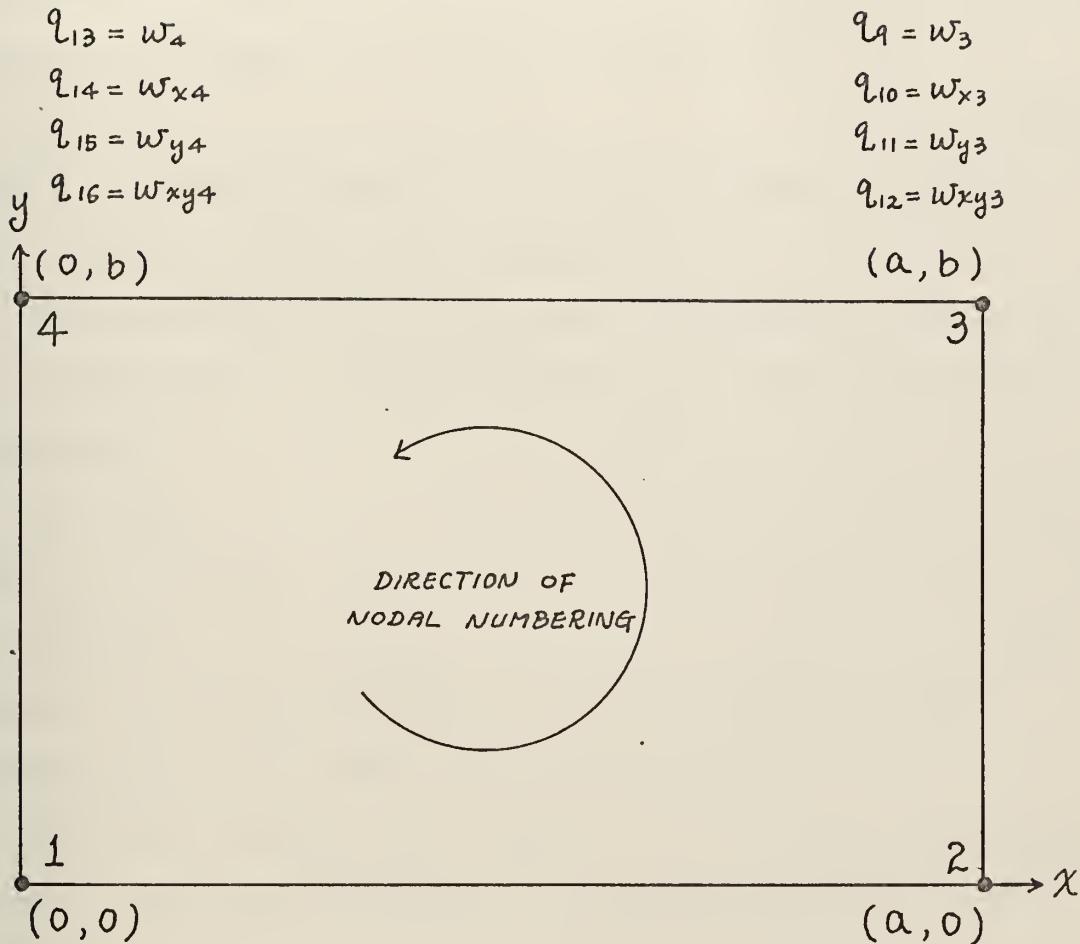


Figure 5 Local Coordinate of a Two dimensional Rectangular Finite Element and Generalized Displacement Numbering

From equation (43a-d), the matrix equation can be defined such that at each nodal point, the generalized displacements (deflections, angles in x and y, and twisting angles) are expressed in terms of the geometry of the element and the some unknown coefficients C_i , which is the form of,

$$\underline{\underline{q}} = \underline{\underline{T}} \underline{\underline{C}} \quad (44)$$

where $\underline{\underline{T}}$ is 16 by 16 matrix, $\underline{\underline{C}}$ is 16 by 1 matrix, and $\underline{\underline{q}}$ is 16 by 1 matrix defined as in Appendix A.

From equation (44), the unknown coefficient matrix, $\underline{\underline{C}}$, can be expressed in terms of geometry and the generalized displacements,

$$\underline{\underline{C}} = \underline{\underline{G}} \underline{\underline{q}} \quad (45)$$

where

$$\underline{\underline{G}} = \underline{\underline{T}}^{-1} \quad (46)$$

However, the actual calculation of the inverse of $\underline{\underline{T}}$ is carried out by the computer in this analysis.

From the elastic plate theory,^(8,9,10) the strain energy stored in the plate during the elastic deformation is expressed by

$$U_p = \frac{D}{2} \int_0^a \int_0^b \left\{ \left[\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right]^2 - 2(1-\nu) \left[\frac{\partial^2 w}{\partial x^2} \cdot \frac{\partial^2 w}{\partial y^2} - \frac{\partial^2 w}{\partial x \partial y} \right]^2 \right\} dx dy$$

This can be written in the matrix form,⁽⁷⁾ which is

$$U_p = \frac{1}{2} \int_0^a \int_0^b \underline{\underline{K}}^T \underline{\underline{D}} \underline{\underline{K}} dy dx \quad (47)$$

where

$$\underline{\underline{K}} = - \begin{pmatrix} \frac{\partial^2 w}{\partial x^2} \\ \frac{\partial^2 w}{\partial y^2} \\ 2 \frac{\partial^2 w}{\partial x \partial y} \end{pmatrix} = - \underline{\underline{H}} \underline{\underline{C}} \quad (48)$$

in which $\underline{\underline{H}}$ is given by Appendix A,

$$\underline{\underline{D}}' = \frac{Et^3}{12(1-v^2)} \begin{pmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{pmatrix} = \beta \cdot \underline{\underline{D}} \quad (49)$$

in which

$$\beta = \frac{Et^3}{12(1-v^2)}$$

and

$$\underline{\underline{D}} = \begin{pmatrix} 1 & v & 0 \\ v & 1 & 0 \\ 0 & 0 & \frac{1-v}{2} \end{pmatrix}$$

Therefore, equation (48) becomes after taking the variation with respect to generalized displacement,

$$\begin{aligned} \delta U_p &= \int_0^a \int_0^b (\delta \underline{\underline{K}}^T) \beta \underline{\underline{D}} \underline{\underline{K}} dy dx \\ &= \int_0^a \int_0^b (\delta \underline{\underline{C}}^T) (\underline{\underline{H}}^T \beta \underline{\underline{D}} \underline{\underline{H}} \underline{\underline{C}}) dy dx \\ &= \delta \underline{\underline{q}}^T \beta \underline{\underline{G}}^T \left[\int_0^a \int_0^b \underline{\underline{H}}^T \underline{\underline{D}} \underline{\underline{H}} dy dx \right] \underline{\underline{G}} \underline{\underline{q}} \end{aligned} \quad (50)$$

The welding energy for the elements shown in Figure 6b can be written as,

$$U_w = \int_0^a \frac{c_y}{2} (\theta_o - \theta_y)^2 dx + \int_0^b \frac{c_x}{2} (\theta_o - \theta_x)^2 dy$$

Therefore,

$$U_w = \frac{c_y}{2} \int_0^a (\theta_o - \frac{\partial w}{\partial y})^2 dx + \frac{c_x}{2} \int_0^b (\theta_o - \frac{\partial w}{\partial x})^2 dy \quad (51)$$

which becomes,

$$\begin{aligned} U_w &= \left[\frac{c_y}{2} \int_0^a (\frac{\partial w}{\partial y})^2 dx + \frac{c_x}{2} \int_0^b (\frac{\partial w}{\partial x})^2 dy \right] \\ &\quad - \left[c_y \theta_o \int_0^a \frac{\partial w}{\partial y} dx + c_x \theta_o \int_0^b \frac{\partial w}{\partial x} dy \right] \\ &\quad + \left[\frac{1}{2} (c_y + c_x) \theta_o^2 \right] \end{aligned} \quad (52)$$

and taking the variation,

$$\begin{aligned} \delta U_w &= \left[c_y \int_0^a \left(\frac{\partial \delta w}{\partial y} \right) \left(\frac{\partial w}{\partial y} \right) dx + c_x \int_0^b \left(\frac{\partial \delta w}{\partial x} \right) \left(\frac{\partial w}{\partial x} \right) dy \right] \\ &\quad - \left[c_y \theta_o \int_0^a \left(\frac{\partial \delta w}{\partial y} \right) dx + c_x \theta_o \int_0^b \left(\frac{\partial \delta w}{\partial x} \right) dy \right] \end{aligned} \quad (53)$$

in which the first bracketed term is the additional stiffness matrix due to welding, and the second is equivalent load matrix for the welding. In matrix form, equation (53) can be written, if some matrices defined as below:

$$\frac{\partial w}{\partial x} = \underline{\underline{F}}_y \underline{\underline{C}} = \underline{\underline{C}}^T \underline{\underline{F}}_y^T \quad (54a)$$

$$\frac{\partial w}{\partial y} = \tilde{F}_x \tilde{C} = \tilde{C}^T \tilde{F}_x^T \quad (54b)$$

where the subscripts x and y denote that the integration should be carried out along the x- and y-direction, respectively, and the matrix \tilde{F}_x and \tilde{F}_y are defined in Appendix A.

Therefore,

$$\left(\frac{\partial \delta w}{\partial y} \right) \left(\frac{\partial w}{\partial y} \right) = (\delta \tilde{C}^T) (\tilde{F}_y^T \tilde{F}_y \tilde{C}) = (\delta \tilde{q}^T) \tilde{G}^T \tilde{F}_y^T \tilde{F}_y \tilde{G} \tilde{q} \quad (55a)$$

$$\left(\frac{\partial \delta w}{\partial x} \right) \left(\frac{\partial w}{\partial x} \right) = (\delta \tilde{C}^T) (\tilde{F}_x^T \tilde{F}_x \tilde{C}) = (\delta \tilde{q}^T) \tilde{G}^T \tilde{F}_x^T \tilde{F}_x \tilde{G} \tilde{q} \quad (55b)$$

Using equations (54a-b) and (55a-b), the welding term can be written as:

$$\begin{aligned} \delta U_w &= \delta \tilde{q}^T C_y \tilde{G}^T \left[\int_0^a (\tilde{F}_x^T \tilde{F}_x) dx \right] \tilde{G} \tilde{q} \\ &+ \delta \tilde{q}^T C_x \tilde{G}^T \left[\int_0^b (\tilde{F}_y^T \tilde{F}_y) dy \right] \tilde{G} \tilde{q} \\ &- \delta \tilde{q}^T C_y \theta_o \tilde{G}^T \left[\int_0^a (\tilde{F}_x^T) dx \right] \\ &- \delta \tilde{q}^T C_x \theta_o \tilde{G}^T \left[\int_0^b (\tilde{F}_y^T) dy \right] \end{aligned} \quad (56)$$

As in the one dimensional case already shown, the total variation should be zero to be in equilibrium of this system. Therefore,

$$\delta U_T = \delta U_p + \delta U_w = 0$$

which is from equations (50) and (56),

$$\begin{aligned}
 (\delta \underline{\underline{q}}^T) \left\{ \underline{\underline{G}}^T [\beta \int_0^a \int_0^b \underline{\underline{H}}^T \underline{\underline{D}} \underline{\underline{H}} dy dx + C_Y \int_0^a \underline{\underline{F}}_x^T \underline{\underline{F}}_x dx \right. \\
 \left. + C_X \int_0^b \underline{\underline{F}}_y^T \underline{\underline{F}}_y dy] \underline{\underline{G}} \underline{\underline{q}} - \underline{\underline{G}}^T [C_Y \theta_o \int_0^a \underline{\underline{F}}_x^T \underline{\underline{F}}_x dx \right. \\
 \left. + C_X \theta_o \int_0^b \underline{\underline{F}}_y^T \underline{\underline{F}}_y dy] \right\} = 0
 \end{aligned} \quad (57)$$

These are the 16 by 16 simultaneous equations which have to be solved if only one element is concerned as shown in the one dimensional case, with the proper boundary and symmetry conditions. From equation (57), the stiffness matrix for a plate element,

$$\underline{\underline{k}} = \underline{\underline{G}}^T \beta \int_0^a \int_0^b \underline{\underline{H}}^T \underline{\underline{D}} \underline{\underline{H}} dy dx \underline{\underline{G}} \quad (58a)$$

The additional stiffness matrix due to welding along the x-direction of the plate,

$$\underline{\underline{k}}_{wx} = \underline{\underline{G}}^T C_Y \int_0^b \underline{\underline{F}}_x^T \underline{\underline{F}}_x dx \underline{\underline{G}} \quad (58b)$$

The additional stiffness matrix due to welding along the y-direction of the plate,

$$\underline{\underline{k}}_{wy} = \underline{\underline{G}}^T C_X \int_0^a \underline{\underline{F}}_y^T \underline{\underline{F}}_y dy \underline{\underline{G}} \quad (58c)$$

The equivalent load matrix due to welding along the x-direction of the plate,

$$\underline{\underline{Q}}_x = \underline{\underline{G}}^T C_Y \theta_o \int_0^a \underline{\underline{F}}_x^T dx \underline{\underline{G}} \quad (58d)$$

The equivalent load matrix due to welding along the y-direction of the plate,

$$\underline{\underline{Q}}_y = \underline{\underline{G}}^T C_X \theta_o \int_0^b \underline{\underline{F}}_y^T dy \underline{\underline{G}} \quad (58e)$$

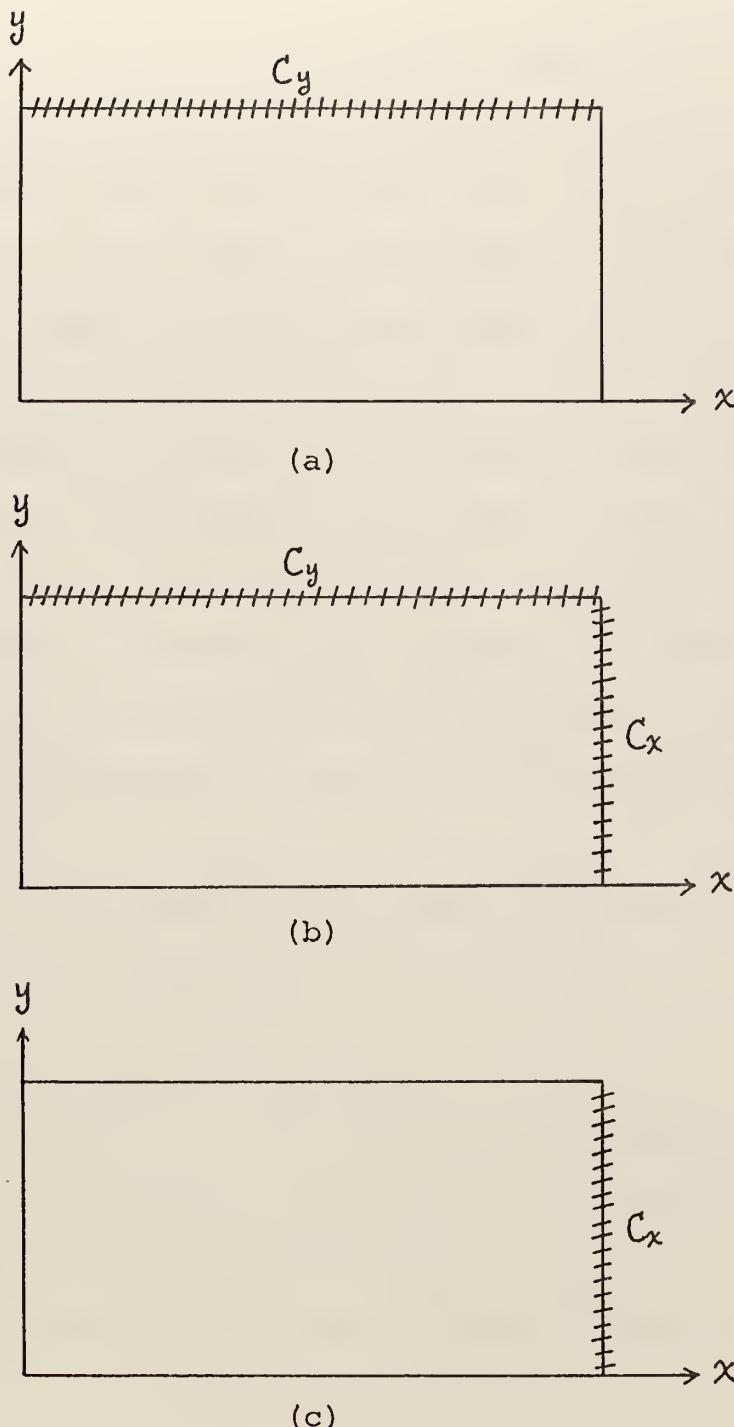


Figure 6 Various Finite Element Having Welded Edge

2. Assembling Procedure

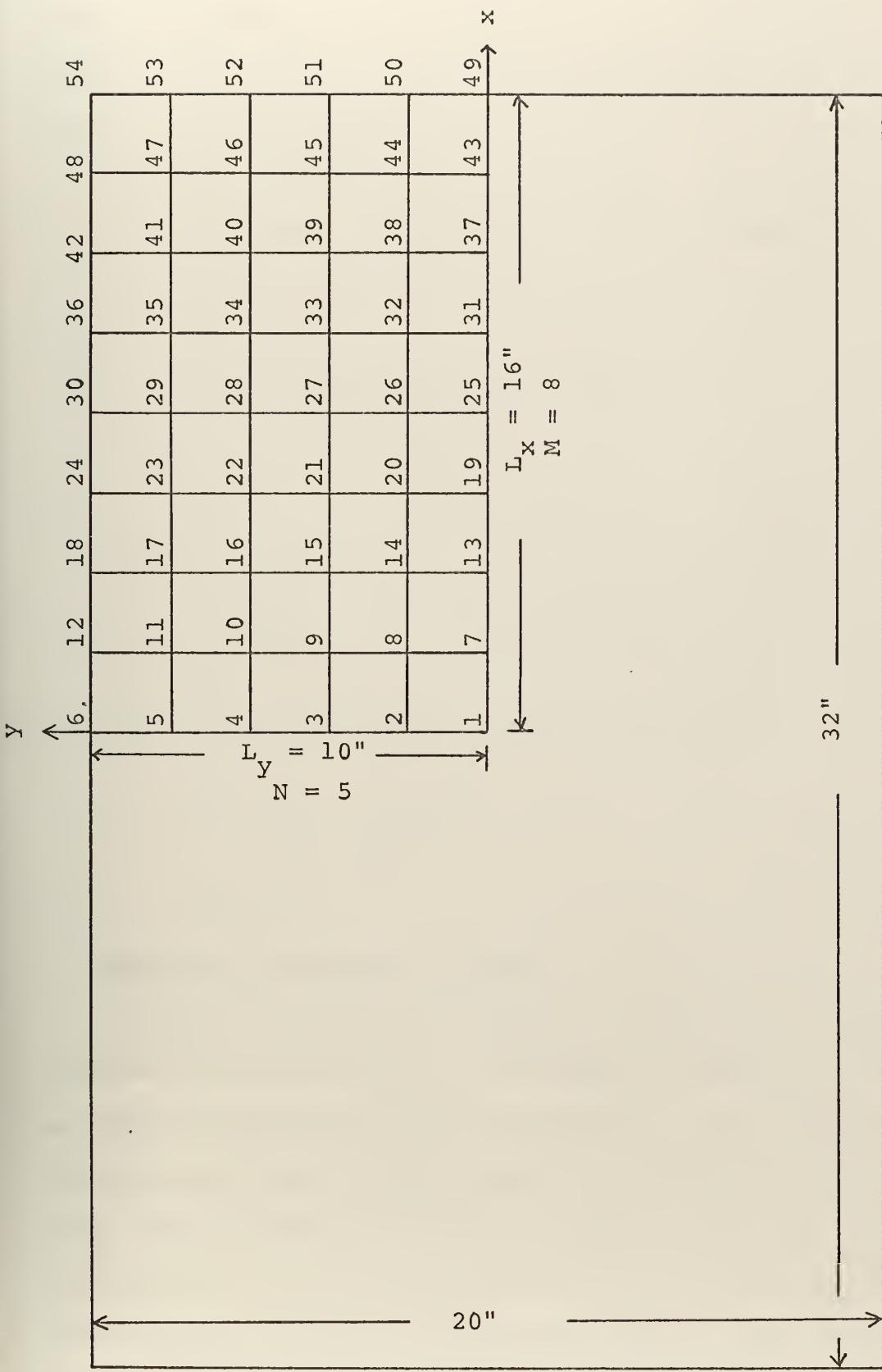
For all the finite elements except those having the welded edge, the stiffness matrix necessary to be assembled is the expression given by equation (58b), and for the finite elements having welded edges, three cases are involved. The first case is elements having welded edges along the x-direction as in Figure 6b; the second case is along the y-direction, as in Figure 6c; and the third case is along x- and y-directions as in Figure 6b. The assembling procedure is the expression of equation (57) for the overall plate which is to be solved, which is described in FEABL user's manual.⁽¹³⁾ Also, from the symmetry condition, which will be described in the following section, only one-quarter part of the plate is actually being considered.

Example of the assembling procedures for the two finite elements is given in Appendix B.

3. Symmetry and Boundary Conditions

After assembling, relevant symmetry and boundary conditions have to be applied to solve the simultaneous equations, as shown in the one dimensional case. Along the welded edge of x and y, the simply supported edge condition requires that all the displacements at that nodal point have to be zero, and for the x-directed edge, the angular changes in the x-direction are not allowed. In just the same way, for y-directed edge, the angular changes in y are not allowed. The symmetry condition requires that the first derivative

Figure 7 Dimensions and Element Meshes of One-quarter of the Plate
for Computer Input



along the symmetric axis should be zero. For example, the symmetry and boundary conditions for the case given by Figure 7 are as follows:

Boundary conditions:

At node 6, 12, 18, 24, 30, 36, 42, 48, 54

$$w = 0$$

$$w_x = 0$$

At node 49, 50, 51, 52, 53, 54

$$w = 0$$

$$w_y = 0$$

Symmetry conditions:

At node 1, 2, 3, 4, 5, 6

$$w_y = 0$$

At node 1, 7, 13, 19, 25, 31, 37, 43

$$w_x = 0$$

At node 1, 6, 49

$$w_{xy} = 0.$$

4. Numerical Integration Method

Usually in the finite element analysis, the integrations involved are carried out by the numerical methods, which can be easily calculated using the computer. For the present analysis, the nine Gaussian points are chosen for the rectangular finite element,^(14,18) as shown in Figure 8c. In actual integration, it is very convenient to use the normalized coordinate system, as will be described below. From Figure 8a and 8b, normalized coordinate system can be defined as

$$x = \frac{a}{2}(X + 1) \quad (59a)$$

$$y = \frac{b}{2}(Y + 1) \quad (59b)$$

and

$$dx = \frac{a}{2} dX \quad (60a)$$

$$dy = \frac{b}{2} dY \quad (60b)$$

Therefore, the arbitrary integration

$$f(a, b) = \int_0^a \int_0^b P(x, y) dx dy \quad (61)$$

becomes in the normalized coordinates,

$$f(a, b) = \frac{ab}{4} \int_{-1}^1 \int_{-1}^1 P\left[\frac{a}{2}(X+1), \frac{b}{2}(Y+1)\right] dX dY \quad (62)$$

Equation (62) can be integrated by numerically using 9-Gaussian points, which become

$$f(a, b) = \frac{ab}{4} \sum_{i=1}^3 \sum_{j=1}^3 w_i w_j P(x, y) \quad (63)$$

where

$$x = \frac{a}{2}(X_i + 1) \quad (63a)$$

$$y = \frac{b}{2}(Y_j + 1) \quad (63b)$$

$$w_i: w_1 = 5/9, w_2 = 8/9, w_3 = 5/9 \quad (63c)$$

$$w_j: w_1 = 5/9, w_2 = 8/9, w_3 = 5/9 \quad (63d)$$

$$x_i: x_1 = -\sqrt{15}/5, x_2 = 0, x_3 = \sqrt{15}/5 \quad (63e)$$

$$y_j: y_1 = -\sqrt{15}/5, y_2 = 0, y_3 = \sqrt{15}/5 \quad (63f)$$

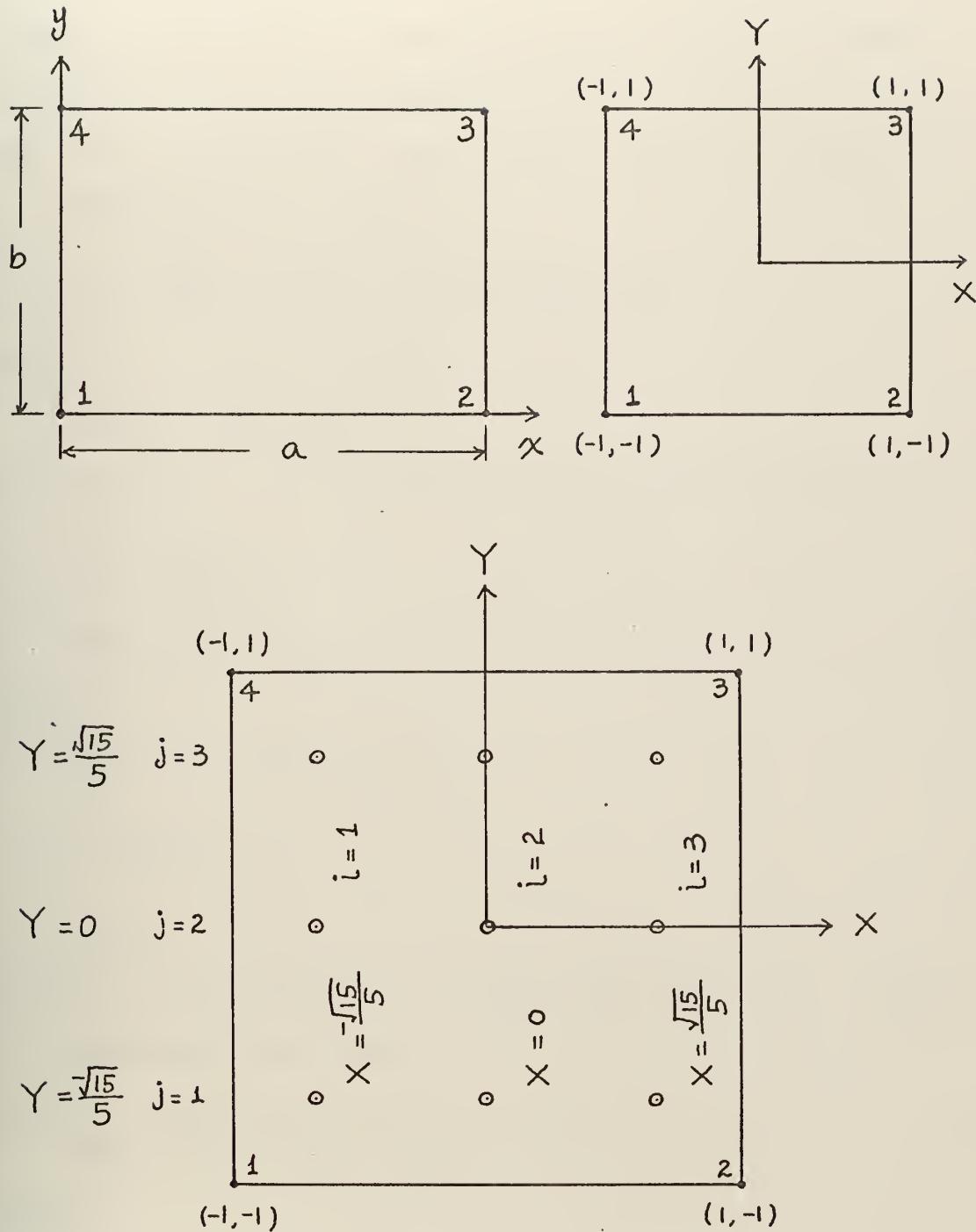


Figure 8 Ordinally and Normalized Coordinates for an Element, and 9 Gaussian Points for Numerical Integration

Therefore, using the concept of normalizing and numerical integration described above, the integrations involved in the present analysis such as equation (58a-e) can easily be written, which are programmed in Appendix D.

From equation (58a)

$$\tilde{k} = \left(\frac{ab\beta}{4} \right) \tilde{G}^T \left[\sum_{i=1}^3 \sum_{j=1}^3 w_i w_j \tilde{P}(x, y) \right] \tilde{G} \quad (64)$$

where

$$\tilde{P}(x, y) = \tilde{H}^T \tilde{D} \tilde{H}$$

x , y , w_i , w_j , x_i and y_j are defined by equation (63a-f).

From equation (58b),

$$\tilde{k}_{wx} = \frac{aC_y}{2} \tilde{G}^T \left[\sum_{i=1}^3 w_i \tilde{P}_x(x, y) \right] \tilde{G} \quad (65)$$

where

$$\tilde{P}_x(x, y) = \tilde{F}_x^T \tilde{F}_x$$

$$x = \frac{a}{2}(x_i + 1)$$

$$y = b$$

w_i and x_i are defined by equation (63c) and (63e), respectively.

From equation (58c),

$$\tilde{k}_{wy} = \frac{bC_x}{2} \tilde{G}^T \left[\sum_{j=1}^3 w_j \tilde{P}_y(x, y) \right] \tilde{G} \quad (66)$$

where

$$\tilde{P}_y(x, y) = \tilde{F}_y^T \tilde{F}_y$$

$$x = a$$

$$y = \frac{b}{2}(y_j + 1)$$

w_j and x_j are defined by equations (63d) and (63f), respectively.

From equation (58d),

$$\underline{Q}_x = \frac{a\theta_o C}{2} \underline{G}^T [\sum_{i=1}^3 w_i \underline{P}_{\ell x}(x, y)] \underline{G} \quad (67)$$

where

$$\underline{P}_{\ell x}(x, y) = \underline{F}_x^T$$

$$x = \frac{a}{2}(x_i + 1)$$

$$y = b$$

w_i and x_i are defined by equation (63c) and (63), respectively.

From equation (58e),

$$\underline{Q}_y = \frac{b\theta_o C}{2} \underline{G}^T [\sum_{j=1}^3 w_j \underline{P}_{\ell y}(x, y)] \underline{G} \quad (68)$$

where

$$\underline{P}_{\ell y}(x, y) = \underline{F}_y^T$$

$$x = a$$

$$y = \frac{b}{2}(y_j + 1)$$

w_j and y_j are defined by equations (63d) and (63f).

As noticed above, numerical integration using the normalized coordinate can simplified the computing procedure by factoring out a and b.

5. Computer Programs

Computations of the ordinally, additional welding, and equivalent welding load matrix of equation (58a-e) have been programmed by using the numerical integration technique with normalized coordinates as listed in Appendix D. In addition to these, FEABL Programs⁽¹³⁾ in assembling of the relevant matrices and solving the simultaneous equations with proper boundary and symmetry conditions are also used and listed in Appendix D.

The Programs listed in Appendix D are only relevant to the panel structure welding with rectangular finite elements. Further, the nodal numbering system should follow the way given by the sample analysis, which means that the numbering should start from origin to y-direction, as shown in Figure 7.

Input data for this analysis are the overall dimensions of one panel structure in the x- and y-direction, thickness of the plate, Young's modulus of elasticity, Poisson's ratio, number of elements divided in x- and y-direction, equivalent welding moment in x- and y-direction, and free joint angular change at the edge. The input data cards are described in Appendix C.

Outputs are the displacements, angular changes in x- and y-directions, and the twisting angles at each nodal point, as shown in Appendix E.

One thing to be noticed here is that for a quarter part of the plate, as shown in Figure 9, the proper boundary and symmetry conditions at the nodal points are to be generated

by subroutine HOLD given in Appendix D; therefore, as long as boundary conditions of simply supported and symmetry conditions as described are concerned, it is not necessary to be considered in the input. Therefore, no matter how the number of finite elements are to be chosen, the only input data will be as described in equation (69a-h). Furthermore, to use this program, the coordinate and a quarter part of the plate should be chosen as the same way as shown in Figure 9.

The units used in the sample program are pounds, inches, and radians. As results, for the output data, the deflections are in inches, the first derivatives (angular changes) are in radians, and the second derivatives with respect to x and y (twisting angles) are also in radians.

Program language used is FORTRAN IV, and the computations are carried out by an IBM 370/155 system.

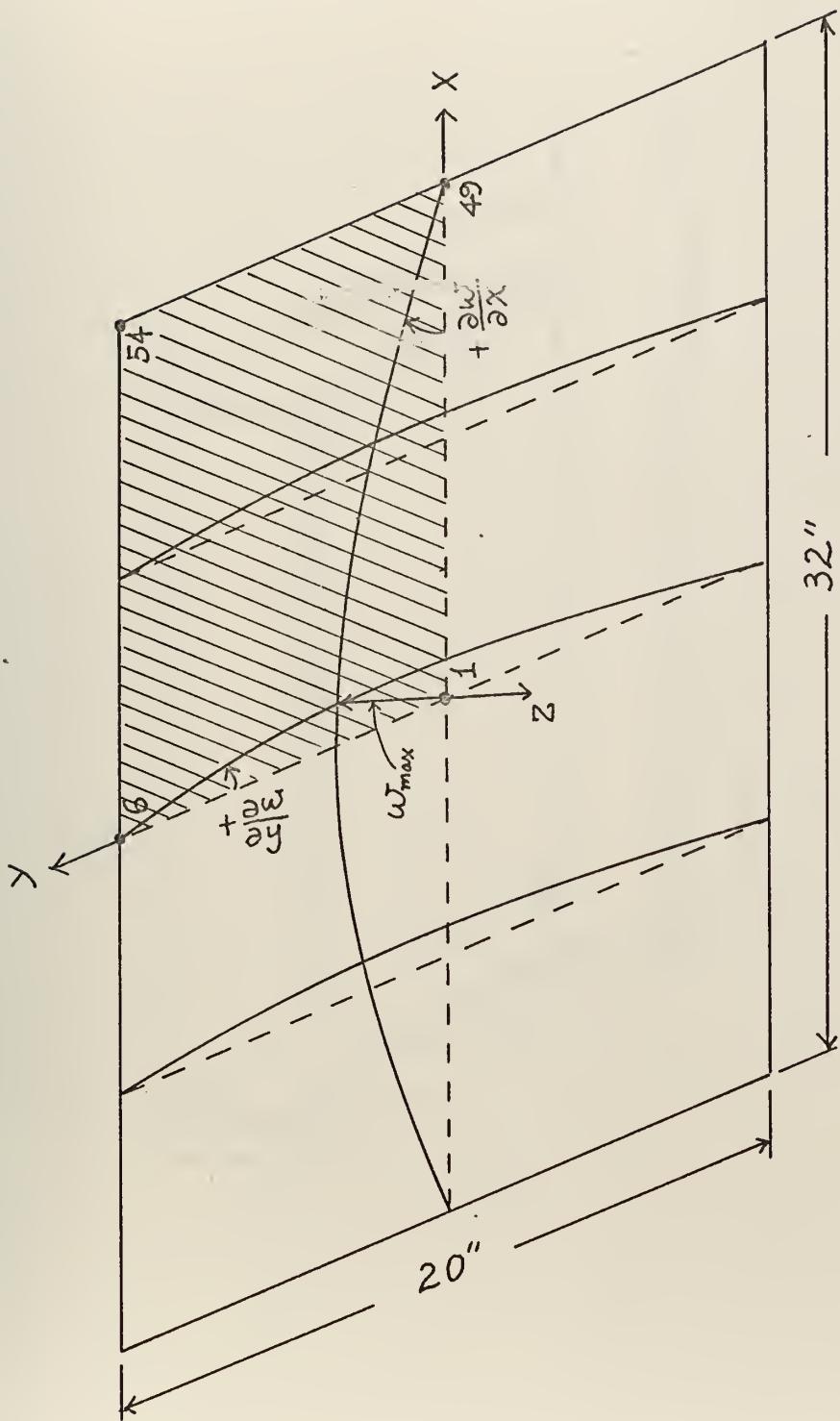


Figure 9 Global Coordinate System and Shape of Welded Deflection

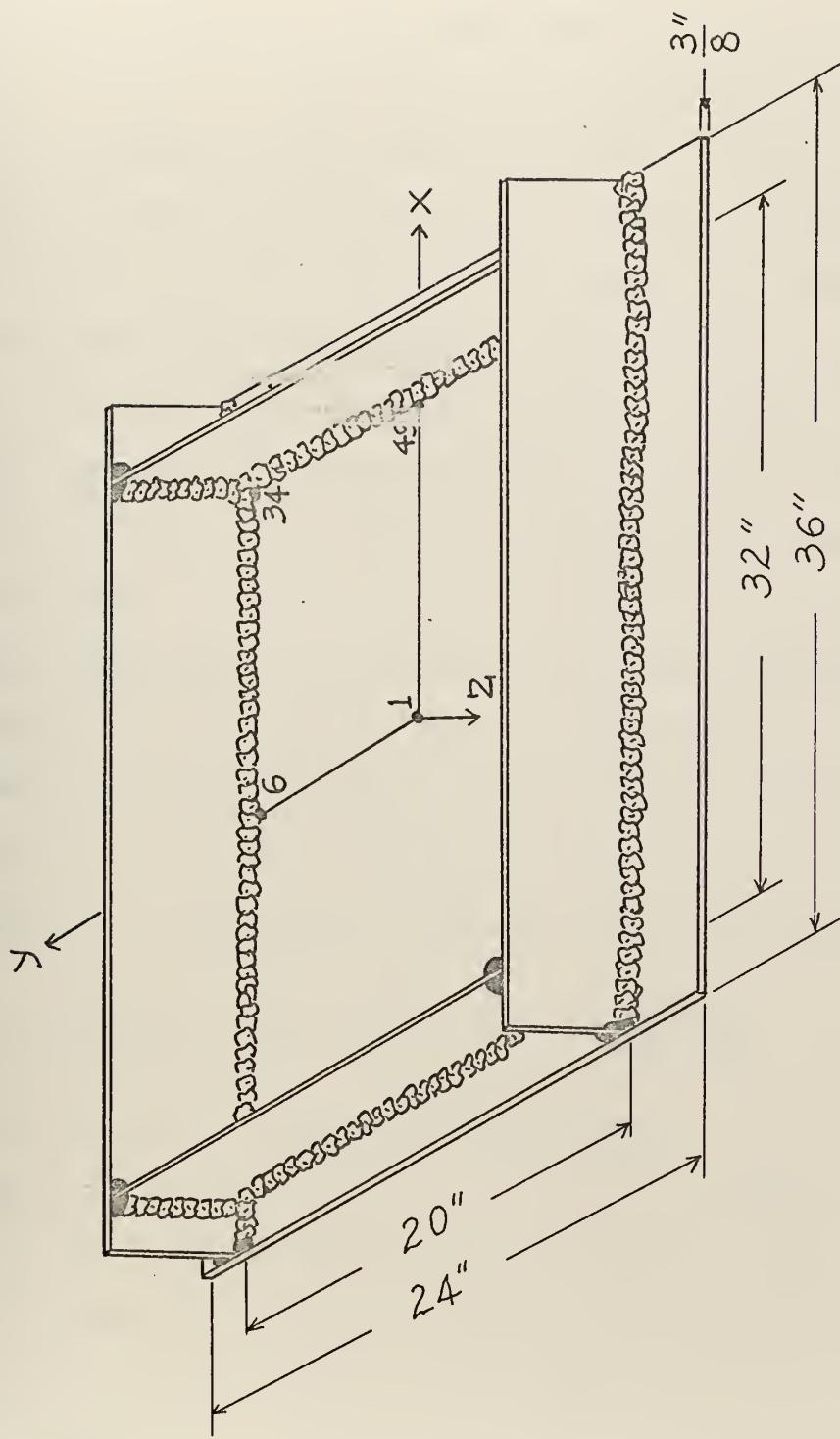


Figure 10 A Panel Structure Analyzed by Experiment and Finite Element Method

III RESULTS

As computer inputs, one steel panel structural dimension, as shown in Figure 10, for which the experiment conducted and weight of consumed welding rods are given, 3 gr/cm.,^(4,19) are used to predict the deformations, angular changes, and twisting angles at the nodal points. Here, the value of the one dimensional free joint angular change along the welded edge, θ_0 , has been read from the results of Figure 3 and the equivalent constrained welding moment, C, is being evaluated using equation (8). The numerical values of the input data are as follows:

Half-length of the plate in the x- and y-directions, respectively, $L_x = 16$ inches and $L_y = 10$ inches.

Number of finite elements in the x- and y-directions, respectively, $M = 8$ and $N = 5$.

Equivalent constrained welding moment from equation (8)

$$C = \frac{t^4}{1 + W/5} = \frac{(25.4 \times 3/8)^4}{1 + 3/5} = 5200 \frac{\text{kg-mm}}{\text{mm rad.}}$$

where W is given by 3 gr/cm from experiment.

Therefore,

$$C = 1.13 \times 10^4 \text{ lb-in/in rad.}$$

Poisson's ratio:

$$\nu = 0.3$$

Young's modulus:

$$E = 30 \times 10^6 \text{ psi.}$$

Thickness of the plate:

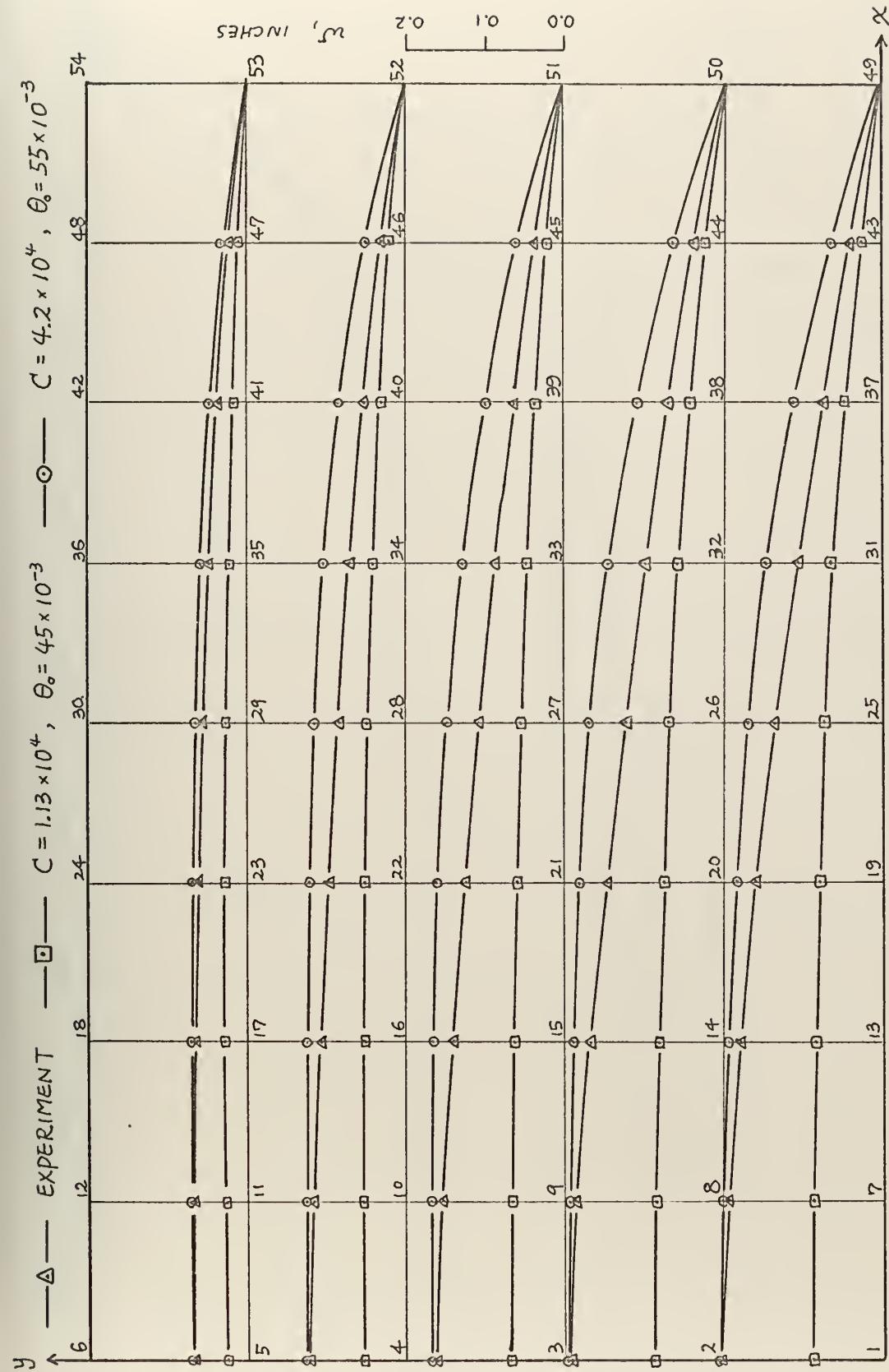
$$t = 3/8 \text{ inch.} \quad (69 \text{ a-h})$$

The results of the analysis using the above data are listed in Appendix E, and these results are plotted in the Figures 11 and 12 to be compared with experimental results. As can be noticed from Figures 11 and 12, the finite element results are lower value than the experiments, and the maximum deflection at the midpoint of the plate which is the most important deflection in reality is only a half of that from the experiment.

To visualize the behavior of the deformations as functions of θ_0 and C, series of computations are conducted with different combinations of θ_0 and C as shown in Table 1. Yet remaining, the rest of the input data are unchanged, and the maximum deflections at nodal point 1 are plotted in terms of θ_0 and C as in Figures 13 and 14, and the maximum angular changes at node 6 and 49 are plotted in the Figures 15 and 16. Only the maximum deflections at node 1, and the maximum angular changes at node 6 and 49 from the results of series computations are tabulated in Tables 2 and 3.

Finally, computed results using $\theta_0 = 55 \times 10^{-3}$ and $C = 4.2 \times 10^4$ are listed in Appendix E, and also plotted in Figures 11 and 12 to compare with the experiment. As can be seen from Figures 11 and 12, these results are very close to the experiment and the reason of choosing the above θ_0 and C will be discussed in the following section.

Figure 11 Deflection Comparison with Experiment and Finite Element Analysis



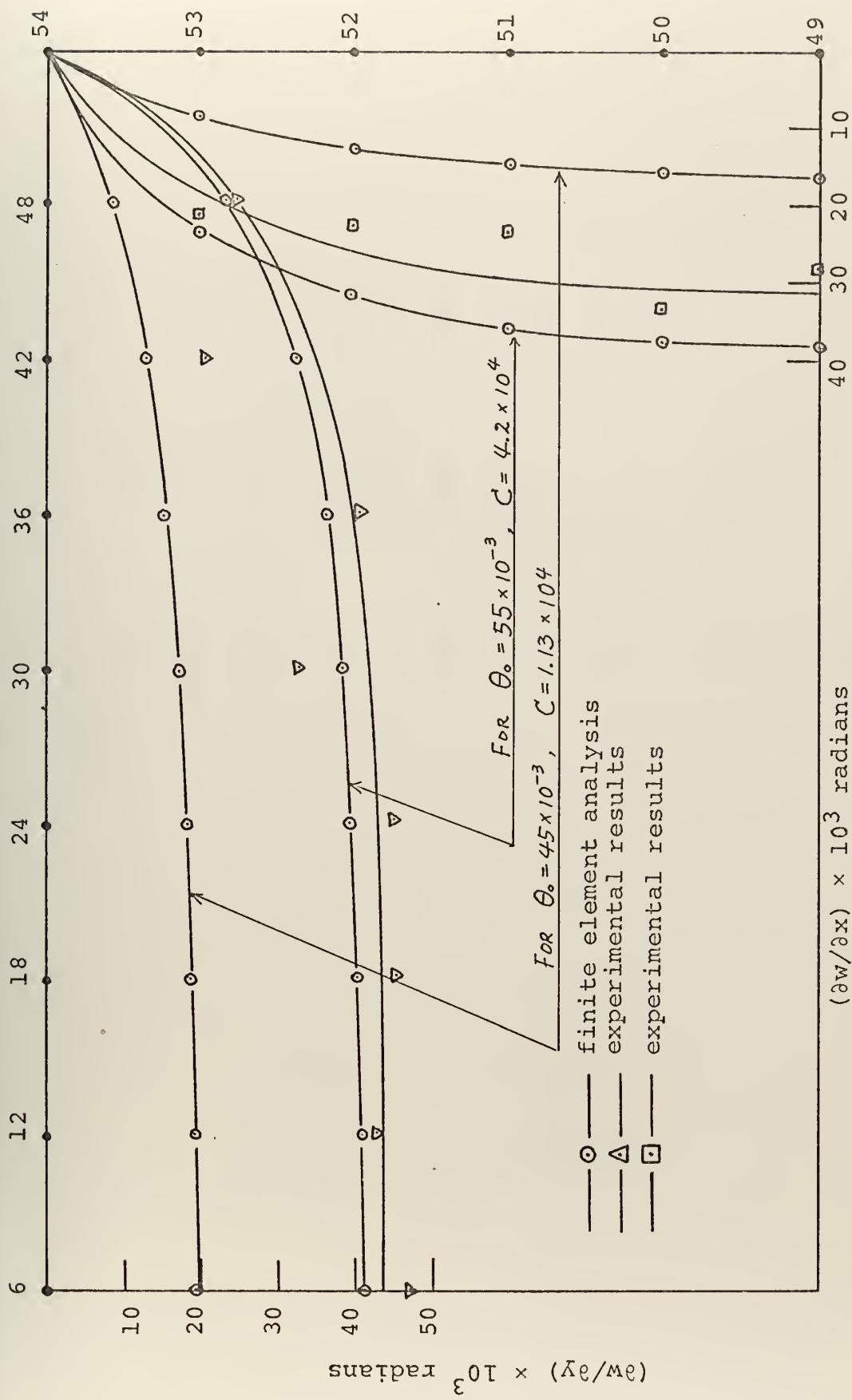


Figure 12 Angular Changes along the Welded Boundary Compared with Experiment and Finite Element Analysis

Table 1

Maximum Deflection at Node 1 in Inches
with Different Combinations of C and θ_o

θ_o	C	1.13×10^4	4×10^4	8×10^4	11×10^4
45×10^{-3}		0.092	0.166	0.196	0.206
55×10^{-3}		0.112	0.203	0.240	0.252
65×10^{-3}		0.133	0.240	0.283	0.298
75×10^{-3}		0.153	0.277	0.327	0.343
85×10^{-3}		0.174	0.314	0.370	0.389
95×10^{-3}		0.194	0.351	0.414	0.435

Table 2

Maximum Angular Changes ($\partial w / \partial y$) at Node 6
 in 10^3 radians with Different Combinations of C and θ_o

θ_o	C	1.13×10^4	4×10^4	8×10^4	11×10^4
45×10^{-3}	18.86	32.28	38.67	40.33	
55×10^{-3}	23.05	40.68	47.26	49.30	
65×10^{-3}	27.23	48.07	55.85	58.26	
75×10^{-3}	31.43	55.47	64.45	67.23	
85×10^{-3}	35.62	62.86	73.04	76.29	
95×10^{-3}	39.81	70.26	81.63	85.16	

Table 3

Maximum Angular Changes ($\partial w / \partial x$) at Node 49
 in 10^3 radians with Different Combinations of C and θ_o

θ_o	C	1.13×10^4	4×10^4	8×10^4	11×10^4
45×10^{-3}	16.59	31.00	37.18	39.22	
55×10^{-3}	20.28	37.90	45.45	47.94	
65×10^{-3}	23.97	44.79	53.71	56.66	
75×10^{-3}	27.66	51.68	61.97	65.38	
85×10^{-3}	31.35	58.57	70.24	74.09	
95×10^{-3}	35.04	65.46	78.50	82.81	

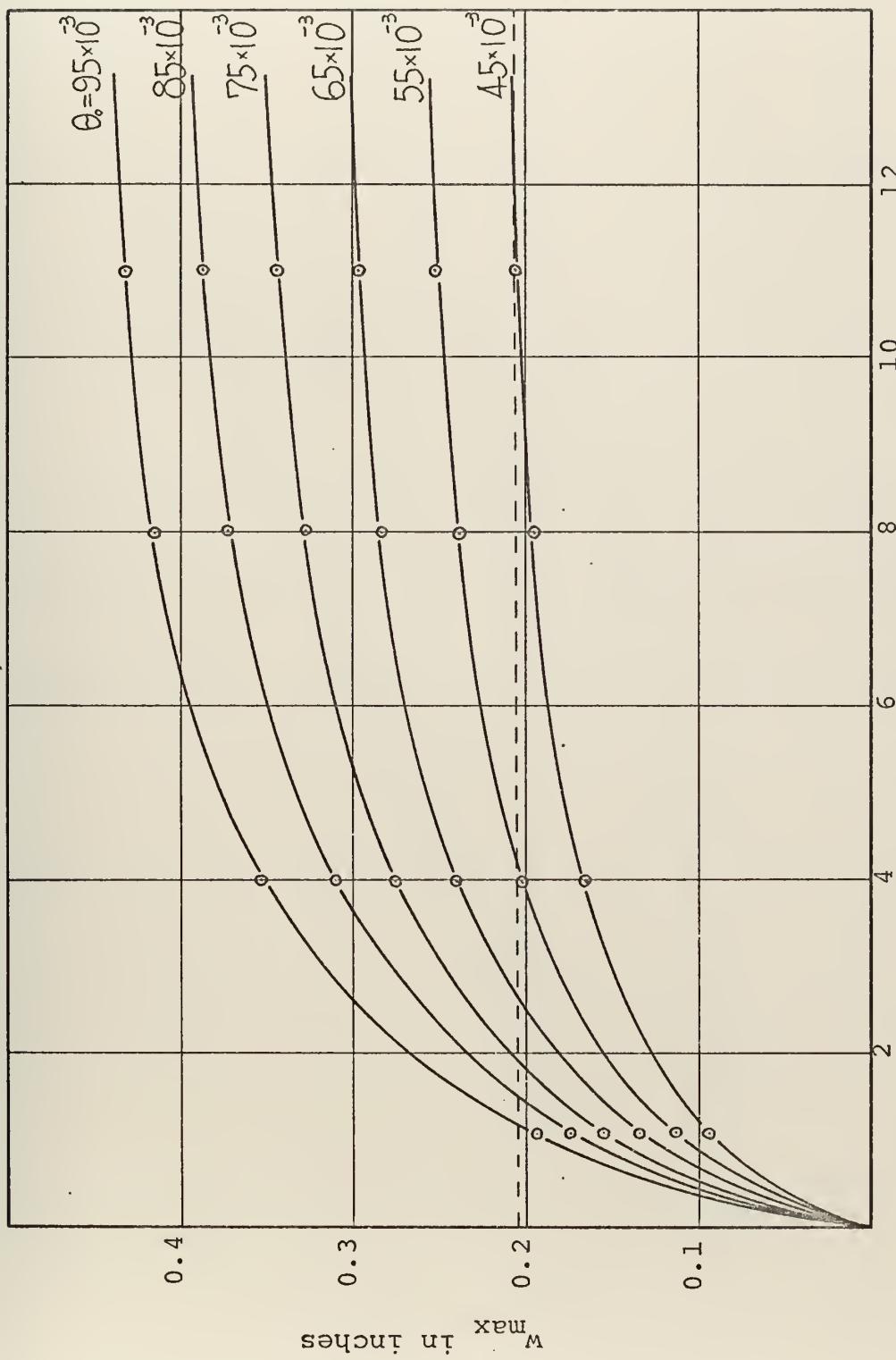


Figure 13 w_{max} at Node 1 versus C with Different θ_0 , Dashed Line Represents Experimental Value.

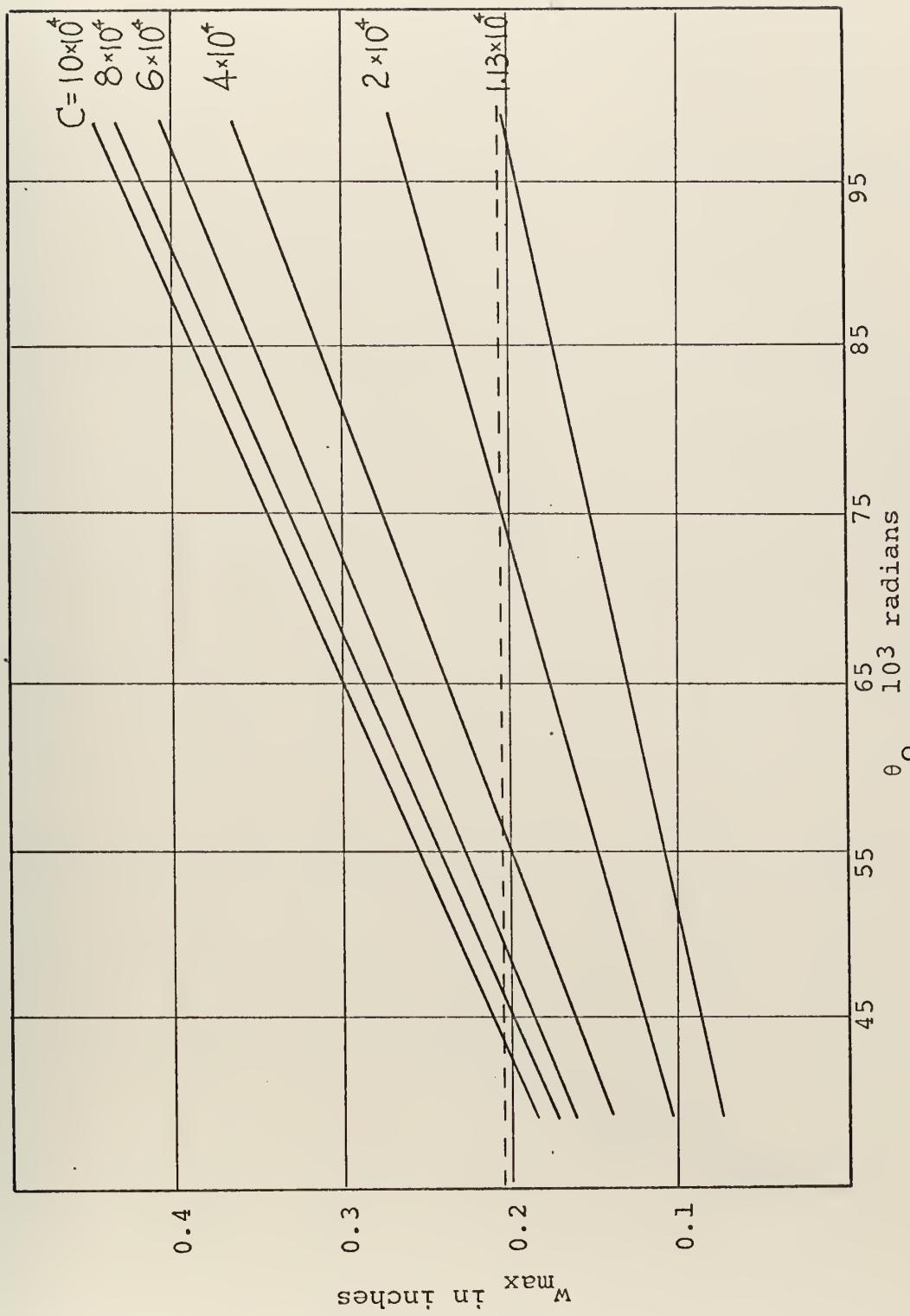


Figure 14 w_{max} at Node 1 versus θ_0 with Different C , Dashed Line Represents Experimental Value.

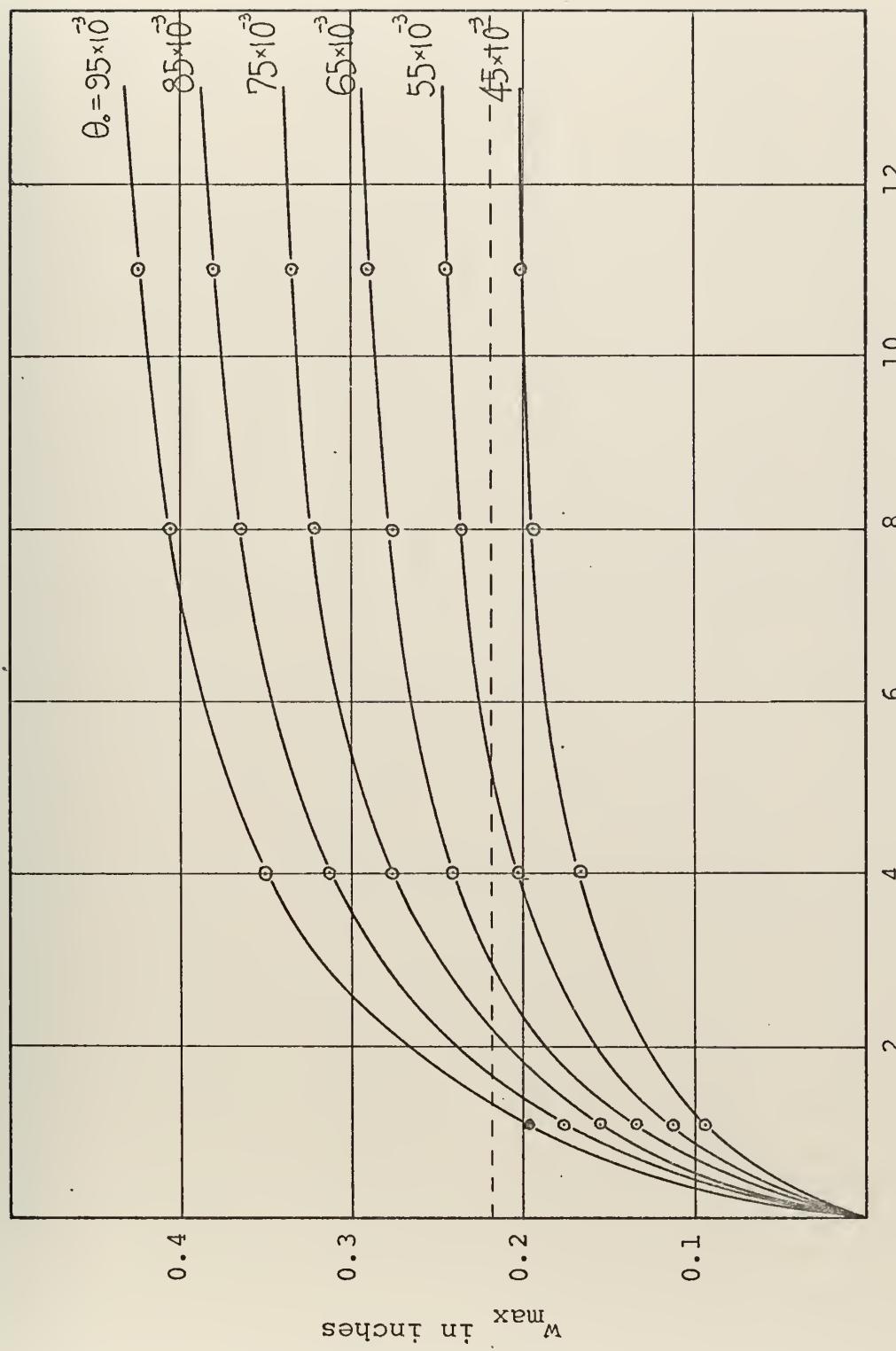


Figure 15 $(\partial w / \partial y)_{\max}$ at Node 6, Dashed Line Represents Experimental Value

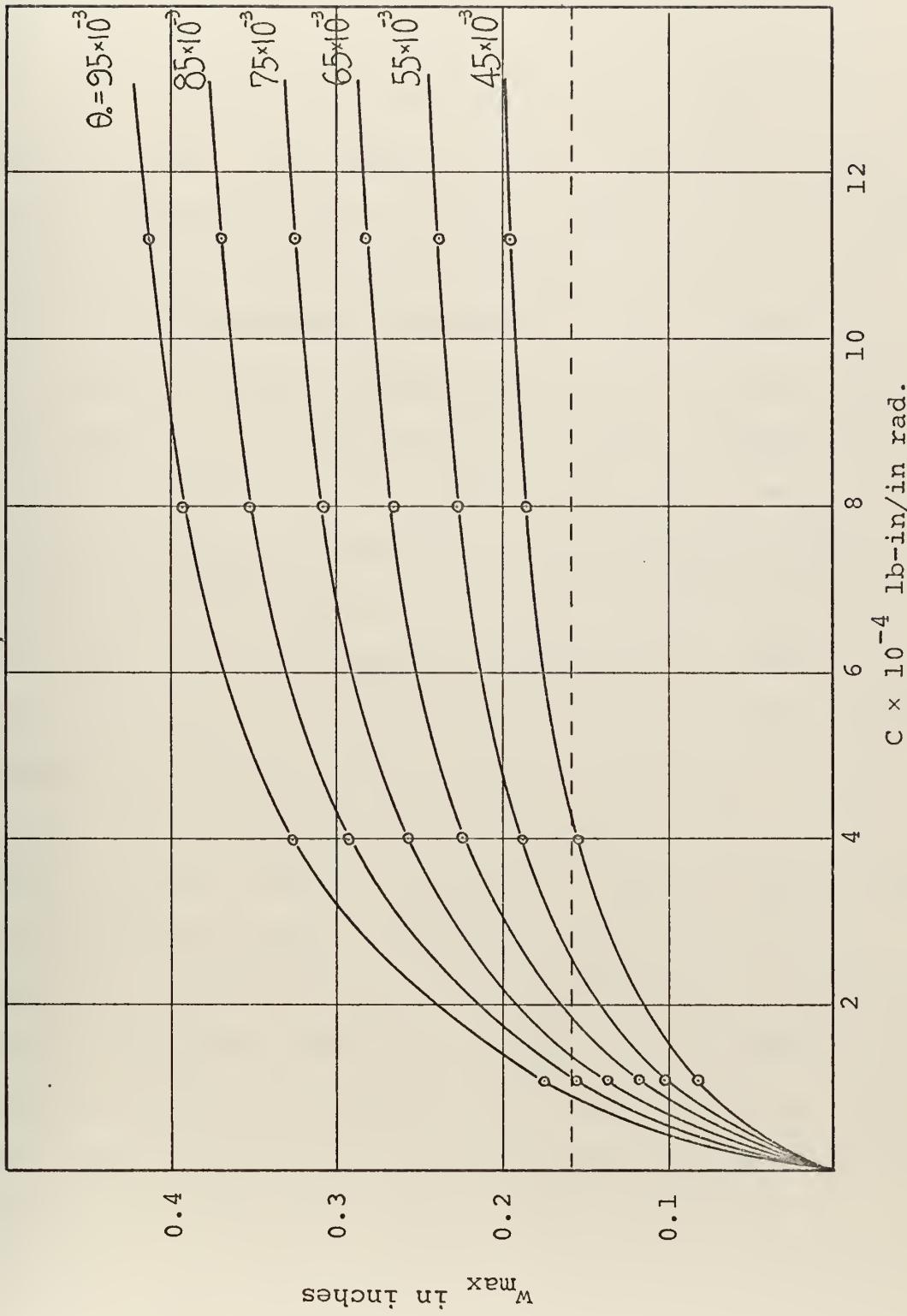


Figure 16 $(\partial w / \partial x)_{\max}$ at Node 49, Dashed Line Represents Experimental Value

IV DISCUSSION OF RESULTS

As has been mentioned in the previous section, the results using $C = 1.13 \times 10^4$ and $\theta_0 = 45 \times 10^{-3}$ are only a half of the experimentally measured values as shown in Figures 11 and 12. The reasons of this difference are not yet known. These may result from the assumption of the elastic behavior of the welded deformation, or from the uncertainty of the experimental measurement itself because the deflections and angular changes which are only of the order of magnitude of 10^{-1} inches and 10^{-2} radians, respectively. Yet, other possibilities are from the value of θ_0 and C .

To investigate the behavior of the deformation with respect to θ_0 and C , series of computations have been conducted. The plotting of maximum deflections at node 1 versus C with changing θ_0 as in Figure 13 shows some insight of deflection behavior with varying C . In here, it is obvious that the deflections are more rapidly changed with changing θ_0 than C changes, which means that the deflections are more strongly dependent function of θ_0 rather than C . Furthermore, the deflection reaches rapidly to an asymptotic value as C increases, which can be seen clearly from the one dimensional deflection, equation (41) where the term $2EI/lc$ becomes small compared to 1 as C increases; therefore, in the limit case, the deflection can be expressed only by $w = l\theta_0/4$.

This behavior of the deflection with changing C are clearly shown in Figure 13. Also from Figure 14, in which the maximum deflections are plotted with respect to θ_o , it is obvious that the deflections are rapidly increased as θ_o increases.

If assumed that the difference between experiment and computed results are caused from the possibility of different values of θ_o and C, then the question is how to choose θ_o and C, such that the computed maximum deflection becomes the same to that from the experiment. From Figures 12 and 13, the combinations of θ_o and C to satisfy the given value of maximum deflection from the experiment can be predicted. However, the best combination is not known, but for the known experiment value of maximum deflection it may be safe to choose θ_o and C such that the changes of both θ_o and C from the calculated value be minimized. In other words, superposing Figures 13 and 14 as shown in Figure 17, the point which gives the same reading of θ_o and C in both ways as shown by the small circle, which is read as $\theta_o = 55 \times 10^{-3}$ and $C = 4.2 \times 10^4$, will be the optimal combination to satisfy the given maximum deflection. The comparison of the results using this θ_o and C to the experiments, as in Figures 11 and 12, shows that this approach gives reasonably good results.

As has been seen from above, there exists many uncertainties for the two dimensional welding analysis, but the welding deformation phenomena can be predicted with a reasonable degree of accuracy by the finite element analysis.

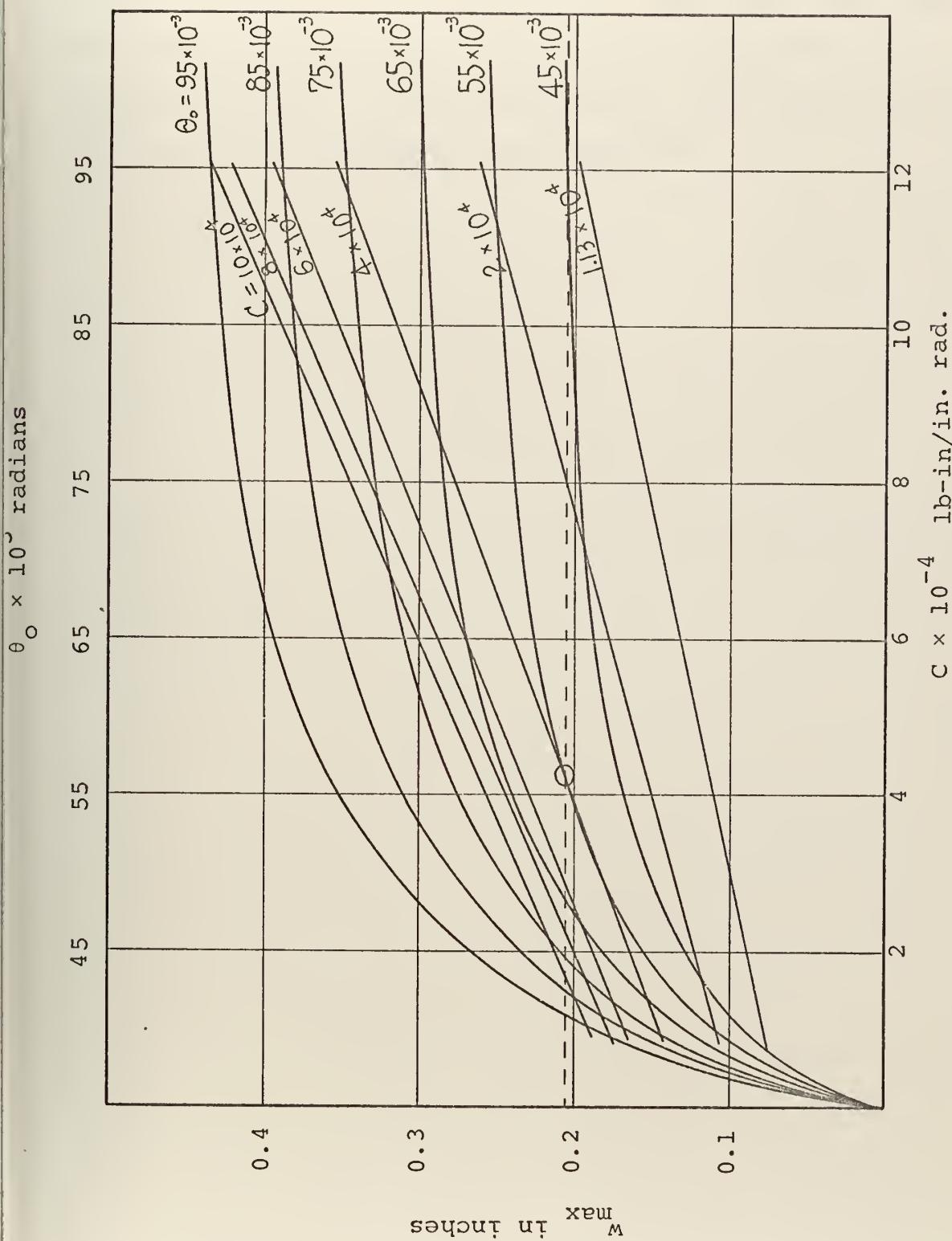


Figure 17 w_{\max} at node 1 after superposition of Figures 13 and 14,
dashed line represents experimental value.

Furthermore, the empirical formula of C given by equation (8) may not be simply applied to the two dimensional case, and also does the value of θ_0 . If it is so, measuring only the maximum deflection in future experiments, the best combination of θ_0 and C can be predicted as discussed.

V CONCLUSIONS AND RECOMMENDATIONS

A. Conclusions

From the discussion of the previous section, it can be concluded as:

1. The finite element analysis for the two dimensional welding deformations which are assumed to be elastic deformation gives reasonable prediction of deflections.
2. The searching technique of optimal combination of θ_0 and C for a given deflection can be used as shown already, if there exists the uncertainty of θ_0 and C.
3. In the future experiments, this analysis gives the order of magnitude feeling of the deflection measured after welding. It is extremely difficult to measure correct deflections in experiments because the order of magnitude to be measured is very small; therefore, the inherent uncertainties always exist.
4. The magnitude of importance of the welding deformation variables such as θ_0 and C are clearly seen, therefore, more care should be concentrated in measuring θ_0 rather than C.
5. Simple empirical formula for C given by equation (8) may not cover the wide range of the values of C; furthermore, as can be derived from equation (41) it may be logical to express C in terms of maximum deflection, free joint angular change, and rigidity,

such as:

$$C = \theta_o / 4w_{max} - 2EI/l \quad (70)$$

in which θ_o is a function of a given welding condition and w_{max} is the constrained maximum deflection at midpoint of the span for the same condition of the welding as of θ_o . Further, the elastic rigidity and plate dimension terms are expressed by the term of $2EI/l$.

B. Recommendations

In this analysis, the assumption used is the elastic behavior of deformations. As stated in the conclusion, it is still believed reasonable to use the elastic theory, but for more accurate prediction, local plastic deformation phenomena should be included in the analysis.

If the elastic assumption is to be used, more careful analysis to evaluate the value of θ_o and C are recommended in future research. Also, a more rational approach to express C is recommended.

In future research, it is recommended that series of experiments be conducted to measure the maximum plate deformation only, and express C and θ_o in terms of welding and plate dimension. In this way, the computer inputs are always available for any prediction of deformation for the two dimensional panel structures.

APPENDIX A

Definitions of Matrices

$$\begin{array}{c|c|c}
 \left(\begin{array}{c} c_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \\ c_6 \\ c_7 \\ c_8 \\ c_9 \\ c_{10} \\ c_{11} \\ c_{12} \\ c_{13} \\ c_{14} \\ c_{15} \\ c_{16} \end{array} \right) & \stackrel{\text{q}}{\sim} = & \left(\begin{array}{c} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \\ q_7 \\ q_8 \\ q_9 \\ q_{10} \\ q_{11} \\ q_{12} \\ q_{13} \\ q_{14} \\ q_{15} \\ q_{16} \end{array} \right) = \left(\begin{array}{c} w_1 \\ w_{x_1} \\ w_{y_1} \\ w_{xy_1} \\ w_2 \\ w_{x_2} \\ w_{y_2} \\ w_{xy_2} \\ w_3 \\ w_{x_3} \\ w_{y_3} \\ w_{xy_3} \\ w_4 \\ w_{x_4} \\ w_{y_4} \\ w_{xy_4} \end{array} \right)
 \end{array}$$

$$\begin{aligned}
 & \left\{ \begin{array}{ccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & a & 0 & 0 & a^2 & 0 & a^3 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2a & 0 & 3a^2 & 0 & 0 \\ 0 & 0 & 1 & a & 0 & 0 & a^2 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2a & 0 & 0 \\ 0 & 0 & 0 & b & ab & a^2b & a^3b^2 & ab^3 & a^2b^2 \\ 1 & a & b & ab & a^2b^2 & a^3b^3 & a^2b^2 & a^3b^3 & a^3b^3 \\ 0 & 1 & 0 & b & 2a & 0 & 3a^2 & 0 & 0 \\ 0 & 0 & 1 & a & 0 & 2b & 0 & a^2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2a & 0 & 0 \\ 1 & 0 & b & 0 & 0 & b^2 & 0 & 0 & b^3 \\ 0 & 1 & 0 & b & 0 & 0 & b^2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2b & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 2b & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{array} \right\} \\
 & \tilde{\mathbb{T}} = \left[\begin{array}{ccccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & a & 0 & 0 & a^2 & 0 & a^3 & 0 & 0 \\ 0 & 1 & a & 0 & 2a & 0 & 3a^2 & 0 & 0 \\ 0 & 0 & 1 & a & 0 & 0 & 2a & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2a & 0 & 0 \\ 0 & 0 & 0 & b & ab & a^2b & a^3b^2 & ab^3 & a^2b^2 \\ 1 & a & b & ab & a^2b^2 & a^3b^3 & a^2b^2 & a^3b^3 & a^3b^3 \\ 0 & 1 & 0 & b & 2a & 0 & 3a^2 & 0 & 0 \\ 0 & 0 & 1 & a & 0 & 2b & 0 & a^2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2a & 0 & 0 \\ 1 & 0 & b & 0 & 0 & b^2 & 0 & 0 & b^3 \\ 0 & 1 & 0 & b & 0 & 0 & b^2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 2b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2b & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 2b & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{array} \right]
 \end{aligned}$$

$$\underline{H} = \begin{pmatrix} 0 & 0 & 0 & 0 & 2 & 0 & 6x & 2y & 0 & 0 & 6xy & 2y^2 & 0 & 6xy^2 & 2y^3 & 6xy^3 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2x & 6y & 0 & 2x^2 & 6xy & 2x^3 & 6x^2y & 6x^3y \\ 0 & 0 & 2 & 0 & 0 & 0 & 4x & 4y & 0 & 6x^2 & 8xy & 6y^2 & 12x^2y & 12xy^2 & 18x^2y^2 \end{pmatrix}$$

$$\underline{F}_x = [0 \quad 0 \quad 1 \quad x \quad 0 \quad 2y \quad 0 \quad x^2 \quad 2xy \quad 3y^2 \quad x^3 \quad 2x^2y \quad 3xy^2 \quad 2x^3y \quad 3x^2y^2 \quad 3x^3y^2]$$

$$\underline{F}_y = [0 \quad 1 \quad 0 \quad y \quad 2x \quad 0 \quad 3x^2 \quad 2xy \quad y^2 \quad 0 \quad 3x^2y \quad 2xy^2 \quad y^3 \quad 3x^2y^2 \quad 2xy^3 \quad 3x^2y^3]$$

APPENDIX B

Example of Assembling

As an example, the assembling procedure for a quarter part of the plate divided by two finite element shown in Figure 18a will be discussed.

For the element 1, as shown in Figure 18b, the stiffness matrix in a local coordinate can be written from equation (58a, b),

$$\tilde{k}_{t_1} = \tilde{k} + \tilde{k}_{w_x} \quad (A-1)$$

where \tilde{k}_{t_1} is 16 by 16 symmetric matrix. And from equation (58d), the load matrix for the element 1 is,

$$\tilde{Q}_{t_1} = \tilde{Q}_x \quad (A-2)$$

where \tilde{Q}_{t_1} is 16 by 1 matrix. By the same way for element 2, the stiffness and load matrix can be written using equation (58a, b, c) and (58d, e), respectively,

$$\tilde{k}_{t_2} = \tilde{k} + \tilde{k}_{wx} + \tilde{k}_{wy} \quad (A-3)$$

$$\tilde{Q}_{t_2} = \tilde{Q}_x + \tilde{Q}_y \quad (A-4)$$

where \tilde{k}_{t_2} is 16 by 16 symmetric matrix and \tilde{Q}_{t_2} is 16 by 1 matrix.

At every nodal point there are four unknowns (w, w_x, w_y, w_{xy}), therefore, after assembling element 1 and 2, the equations have to be solved with proper boundary and symmetry conditions are 24 by 24 simultaneous equations, which is in the matrix form:

$$\delta \underline{q}^T [\underline{K} \underline{q} - \underline{Q}] = 0 \quad (A-5)$$

where the matrix \underline{K} is 24 by 24 symmetric matrix, \underline{Q} is 24 by 1, \underline{q} is 24 by 1, and \underline{q}^T is 1 by 24 matrix.

All the quantities in equation (A-5) are defined by the global coordinate, therefore, the assembling procedure is to locate the quantities defined by equation (A-1, A-2, A-3, A-4) to the proper position in equation (A-5). This operation can easily be established comparing each nodal points numbered in local coordinates to that of global coordinates:

<u>Element 1, nodal points</u>		<u>Element 2, nodal points</u>	
<u>Coordinates</u>		<u>Coordinates</u>	
local	global	local	global
1	1	1	3
2	3	2	5
3	4	3	6
4	2	4	4

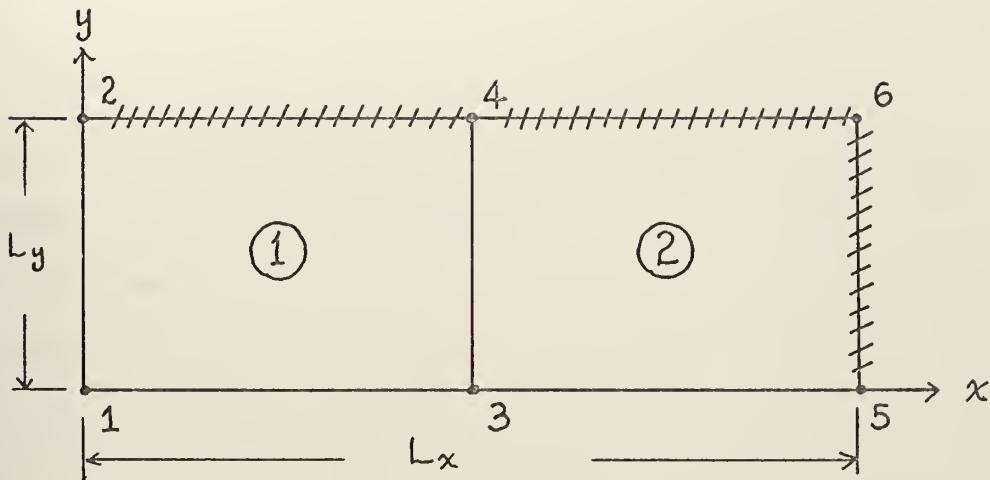
And noticing that numbering of the generalized displacement, q_i , follows the relation,

$$i = 4m - g \quad (A-6)$$

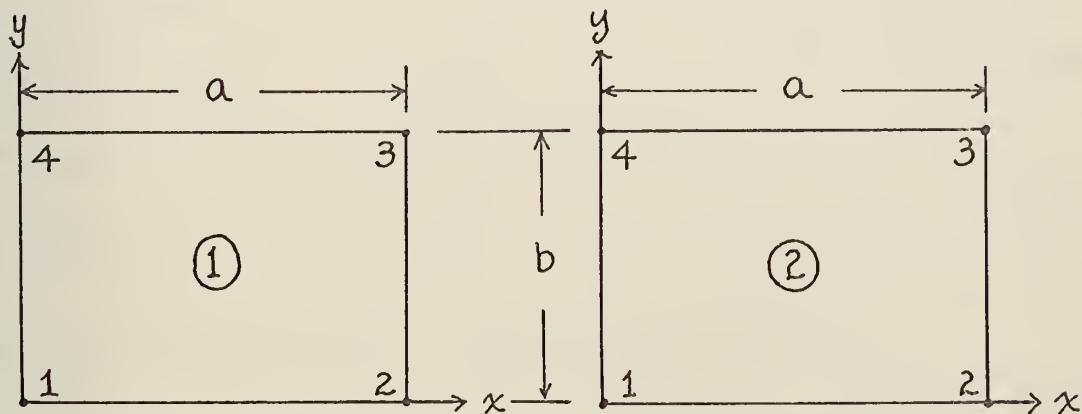
$$j = 4n - h \quad (A-7)$$

where i and j represent the index for generalized displacements, m and n for nodal numbers, g and h are set to be 3 for w , 2 for w_x , 1 for w_y , and 0 for w_{xy} .

For example, one of the stiffness matrix elements at nodal point 4 in global coordinate can be calculated.



a. Nodal points numbering in global coordinate



b. Nodal points numbering in local coordinate

Figure 18 Elements and Nodal Points Numbering in Global and Local Coordinate for Sample Assembling

From equation (A-6) and (A-7),

$$i = 16 - g \quad (A-8)$$

$$j = 16 - h \quad (A-9)$$

But the same node 4 becomes node 3 at element 1 and node 4 at element 2 in local coordinate.

For element 1,

$$(i)_1 = 12 - g \quad (A-10)$$

$$(j)_1 = 12 - h \quad (A-11)$$

and for element 2,

$$(i)_2 = 16 - g \quad (A-12)$$

$$(j)_2 = 16 - h \quad (A-13)$$

Therefore, if $g = 1$ and $h = 2$, then from equation (A-8),

(A-9), and (A-5),

$$K_{ij} = K_{15,14} \quad (A-14)$$

From equations (A-10), (A-11), and (A-1)

$$k_{t_1}_{11,10} \quad (A-15)$$

and from equations (A-12), (A-13), and (A-3),

$$k_{t_2}_{15,14} \quad (A-16)$$

Therefore, element of the stiffness matrix in global coordinate becomes,

$$K_{15,14} = k_{t_1}_{11,10} + k_{t_2}_{15,14} \quad (A-17)$$

All other stiffness matrix element can be expressed by the same manner described above, and the programs of assembling the stiffness matrix are available in FEABL.⁽¹³⁾

APPENDIX C

Description of Input Data

Input data for the programs listed in Appendix D are:

c_x , c_y , θ_o , D , t , E , L_x , L_y , M and N as described in section III. In addition to these, control card for calculation of different cases in each job with changing above input variables are considered. By using this calculation control input card in each job, series of calculation with changing some of the input variables can be calculated, and the deformation behavior with respect to these changing variables can easily be visualized by plotting the deformation versus these variables.

Detailed input data card arrangement and format are listed in Table 4.

Table 4
Input Data Layout Form

<u>Card Column</u>	<u>Input Format</u>	<u>Program Symbol</u>	<u>Definition and Unit</u>	<u>Sample Input</u>
<u>Card Number 1</u>				
1-5	I5	NCASE	Number of cases involved in each job	14
<u>Card Number 2</u>				
1-15	E15.7	LX	Length of 1/4 plate in x, inches	16.0E+00
16-30	E15.7	LY	Length of 1/4 plate in y	10.0E+00
31-35	I5	M	Number of elements in x	8
36-40	I5	N	Number of elements in y	5
<u>Card Number 3</u>				
1-12	E12.7	CX	Equivalent constrained welding moment as in Figure 6, lb-in/in rad.	4.20E+04
13-24	E12.7	CY	Equivalent constrained welding moment as in Figure 6, lb-in/in rad.	4.20E+04
25-36	E12.7	THETAZ	Free joint angular change, θ_0 , radians	55.0E-03
37-48	E12.7	GNU	Poisson's ratio, D	0.30E+00
49-60	E12.7	THK	Plate thickness, t, inches	0.375E+00
61-72	E12.7	E	Young's modulus, E, lb/in. ²	30.0E+06

APPENDIX D

Listing of Programs


```

C DAWPS PROGRAM
C DISTORTION ANALYSIS OF WELDED PANEL STRUCTURE
C FORMULATED BY D. SHIN
C PROGRAMMED BY SUSAN E. FRENCH•••ASPL AT MIT FFR. 1972
C THIS PROGRAM USES FEABL, FINITE ELEMENT ANALYSIS BASIC
C LIBRARY, WRITTEN BY DR. OSCAR ORRINGER OF THE AERONAUTICS AND
C ASTRONAUTICS DEPT. OF MIT
C REAL*4 LX,LY
C DIMENSION REAL(8000),INTGR(8000)
C DIMENSION D(3,3),G(15,16),ELK1(16,16),ELK2(16,16),ELK3(16,16),
1 ELK4(16,16),Q1(16),Q2(16),Q3(16),Q4(16)
C COMMON /ID/ KREAD,KWRITE,KPUNCH
C COMMON /SIZE/ NET, NDT
C COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTER,IQ,IK
C COMMON /END/ LCON,LKOUNT,LLNZ,LMASTER,LQ,LK
C COMMON /CONST/ M,N,A,B,CX,CY,BETA,THETAZ,ABET,HALFA,HALFB,ASQ,ACU
1B,BSQ,BCUB,CXB2,CYA2,BTHCX,ATHCY
C COMMON /GAUSS/ WBAR(3),XBAR(3),YBAR(3)
C COMMON /MESH/ NDE,NNT,LTYP(200)
C EQUIVALENCE (REAL(1),INTGR(1))
C DEFINE LDIM=DIMENSION OF REAL AND INTEGER
LDIM=8000
C DEFINE READ , WRITE AND PUNCH DATA SET REFERENCE NUMBERS
KREAD=5
KWRITE=6
KPUNCH=7
C READ ALL INPUT DATA
READ(KREAD,900) NCASE
900 FORMAT(15)
DO 100 NRUN=1,NCASE
READ(KREAD,1000) LX,LY,M,N,CX,CY,THETAZ,GNU,THK,E
1000 FORMAT(2E15.7,2I5/6E12.7)
C
C LX=LENGTH IN X DIRECTION, LY=LENGTH IN Y DIRECTION, M= NO. MESSES IN X
C DIRECTION, N=NO. OF MESSES IN Y DIRECTION,CX AND CY ARE CONSTANTS,
C THETAZ=THETA SUB ZERO=AN ANGLE IN RADIANS, GNU=NU=POISONS RATIO,

```



```

C THK=SMALL T=THICKNESS, E= YOUNG'S MODULUS
C
C NET=NO. OF ELEMENTS (TOTAL) IN MESH, NDE=NO. OF DEGREES OF FREEDOM
C PFR MESH ELEMENT, NNT=NO. OF NODES (TOTAL) IN MESH, NDT=NO. OF
C DEGREES OF FREEDOM (TOTAL) IN MESH, NCON= NO. OF CONSTRAINTS
C (BOUNDARY CONDITIONS) APPLIED TO TOTAL MESH
C NET=N*N
C
C NDE=16
NNT=(N+1)*(N+1)
NDT=4*NNT
MASTRL=NET*(1+NDE)
NCON=4*(N*M+1)
C CALCULATE SOME CONSTANTS
BETA=E*THK**2/(12.0*(1.0-GNU*GNU))
A=LX/FLOAT(M)
B=LY/FLOAT(N)
HALFA=.5*A
HALFB=.5*B
ASQ=A*A
ACUB=ASQ*A
BSQ=B*B
BCUB=BSQ*B
ABET=A*B*BETA/4.
CXB2=.50*CX*B
CYA2=.5*CY*A
BTHCX=.5*A*THTAZ*CX
ATHCY=.5*A*THTAZ*CY
C CALCULATE T MATRIX, D MATRIX AND (INVERSE OF T) = G MATRIX
CALL TDGMAT(GNU,D,G)
C CALCULATE DATA IN BEGIN AND END LABELLED COMMON
CALL SETUP(LDIM,NCON,MASTRL,REAL,INTGR)
C CALCULATE AND STORE MASTER VECTOR DATA RELATING ELEMENTS AND
C THE DEGREE OF FREEDOM NUMBERS (GLOBAL) AT EACH CORNER
CALL NODDEF(REAL,INTGR)
C FILL IN COUNT AND LNZE SECTIONS OF INTGR AND CALCULATE LK
CALL ORK(LDIM,REAL,INTGR)

```


DFFINE CONSTANTS NEEDED FOR GAUSSIAN QUADRATURE INTEGRATION

```
WBAR(1)=.55555556
WBAR(2)=.88888889
WBAR(3)=.55555556
XBAR(1)=-.7745967
XBAR(2)=0.
```

```
XBAR(3)=+.7745967
YBAR(1)=-.7745967
YBAR(2)=0.
```

```
YBAR(3)=+.7745967
YBAR(4)=+.7745967
```

C CALCULATE ELEMENT STIFFNESS MATRIX FOR NO WELD

```
CALL STIF1(D,G,ELK1,Q1)
```

```
CALL STIFY(G,ELK2,Q2)
```

```
CALL STIFX(G,ELK3,Q3)
```

```
DO 40 KK=1,NDE
```

```
Q2(KK)=Q2(KK)+Q1(KK)
```

```
Q4(KK)=Q2(KK)+Q3(KK)
```

```
Q3(KK)=Q3(KK)+Q1(KK)
```

```
DO 30 LL=1,NDE
```

```
ELK2(KK,LL)=ELK1(KK,LL)+ELK2(KK,LL)
```

```
ELK4(KK,LL)=ELK2(KK,LL)+ELK3(KK,LL)
```

```
ELK3(KK,LL)=ELK3(KK,LL)+ELK1(KK,LL)
```

CONTINUE

40 CONTINUE

C ASSEMBLE TOTAL STIFFNESS MATRIX AND TOTAL Q VECTOR
DO 60 J=1,4
DO 50 LNUM=1,NET

```
IF(LTYPE(LNUM).NE.J) GO TO 50
```

```
GO TO (42,44,46,48),J
```

```
42 CALL ASEMBL(LNUM,NDE,ELK1,Q1,REAL,INTGR)
```

```
GO TO 50
```

```
44 CALL ASEMBL(LNUM,NDE,ELK2,Q2,REAL,INTGR)
```

```
GO TO 50
```

```
46 CALL ASEMBL(LNUM,NDE,ELK3,Q3,REAL,INTGR)
```

```
GO TO 50
```

```
48 CALL ASEMBL(LNUM,NDE,ELK4,Q4,REAL,INTGR)
```



```

50 CONTINUE
60 CONTINUE
C   FIL IN CONSTRAINT VECTOR FOR ENTIRE STRUCTURE
C   CALL HOLDIN,M,ICON,IQ,NCON,IQ,INTGR,REAL)
C   APPLY CONSTRAINTS TO ASSEMBLED K MATRIX
C   CALL BCON(REAL,INTGR)
C   CALL FACTSD(REAL,INTGR)
C   CALL SIMULQ(ENERGY,REAL,INTGR)
C   PRINT ALL INPUT DATA
      WRITE(KWRITE,2000) LX,LY,M,N,CX,CY,THETAZ,GNU,THK,F
2000 FORMAT(1H0/1H0//ODISTORTION ANALYSIS OF WELDED PANEL STRUCTURE•••
1BY D. SHIN'/4H0LX=,E12.5,5H, LY=,E12.5, 23H, M=NJ• ELEMENTS X DIR=,
213/20H0N0. ELEMENTS Y DIR=,I3,5H, CX=,E12.5,5H, CY=,E12.5/9H0THETA
3 0=E12.5,25H RADIANS, POISSEONS RATIO=,E12.5/11H0THICKNESS=,E12.5,
417H, YOUNGS MODULUS=,E12.5)
      WRITE(KWRITE,2120)
2120 FORMAT(5H0NODE,7X,1H#,12X,2HDW,12X,2HDW,8X,10HD( DW/DY )/1H+,23X,4
1H_____,10X,4H_____,6X,12H_____/1H ,24X,24DX,12X,2HDY,12X,2HDX
2)

      ISTART=IQ-4
      DO 80 NO=1,NNT
      ISTART=ISTART+4
      IEND=ISTART+3
      WRITE(KWRITE,2130) NO,(REAL(J),J=ISTART,IEND)
2130 FORMAT(14,4E14.5)
      80 CONTINUE
      100 CONTINUE
      STOP
      END

```


C

```

SUBROUTINE TDGMAT(GNU,D,G)
C CALCULATE T, D AND G=INVERSE OF D MATRICES
COMMON /CONST/ M,N,A,R,CX,CY,BETA,THETAZ,ARBBET,HALFA,HALFR,ASQ,ACU
1B,BSQ,BCJB,CXB2,CYA2,BTHCX,ATHCY
COMMON /IO/ KREAD,KWRITE,KPUNCH
DIMENSION D(3,3),G(16,16),T(16,16),LL(16),MM(16),ALIN(256)
CALL ERASE (T,256)
T(1,1)=1.
T(2,2)=1.
T(2,3)=1.
T(4,4)=1.
T(5,1)=1.
T(5,2)=A
T(5,5)=ASQ
T(5,7)=ACUB
T(6,2)=1.
T(6,5)=2.*A
T(6,7)=3.*ASQ
T(7,3)=1.
T(7,4)=A
T(7,8)=ASQ
T(7,11)=ACUB
T(8,4)=1.
T(8,8)=T(6,5)
T(8,11)=T(6,7)
T(9,1)=1.
T(9,2)=A
T(9,3)=B
T(9,4)=B*A
T(9,5)=ASQ
T(9,6)=BSQ
T(9,7)=ACUB
T(9,8)=ASQ*B
T(9,9)=A*BSQ
T(9,10)=BCUB
T(9,11)=ACUB*B

```



```

T(9,12)=ASQ*BSQ
T(9,13)=A*BCUB
T(9,14)=ACUB*BSQ
T(9,15)=ASQ*BCUB
T(9,16)=ACUB*BCUR
T(10,2)=1.
T(10,4)=B
T(10,5)=T(6,5)
T(10,7)=T(8,11)
T(10,8)=2.*A*B
T(10,9)=BSQ
T(10,11)=3.*T(9,8)
T(10,12)=2.*T(9,9)
T(10,13)=BCUR
T(10,14)=3.*T(9,12)
T(10,15)=2.*T(9,13)
T(10,16)=3.*T(9,15)
T(11,3)=1.
T(11,4)=A
T(11,6)=2.*B
T(11,8)=ASQ
T(11,9)=T(10,8)
T(11,10)=3.*BSQ
T(11,11)=ACU3
T(11,12)=2.*T(9,8)
T(11,13)=3.*T(9,9)
T(11,14)=2.*T(9,11)
T(11,15)=T(10,14)
T(11,16)=3.*T(9,14)
T(12,4)=1.
T(12,8)=T(10,5)
T(12,9)=T(11,6)
T(12,11)=T(10,7)
T(12,12)=4.*T(9,4)
T(12,13)=T(11,10)
T(12,14)=6.*T(9,8)

```



```

T(12,15)=6.*T(9,9)
T(12,16)=9.*T(9,12)
T(13,1)=1.

T(13,3)=B
T(13,6)=BSQ
T(13,10)=BCUB
T(14,2)=1.
T(14,4)=B
T(14,9)=BSQ
T(14,13)=BCUB
T(15,3)=1.
T(15,6)=T(11,6)
T(15,10)=T(11,10)
T(16,4)=1.
T(16,9)=T(15,6)
T(16,13)=T(15,10)
D(1,1)=1.
D(2,1)=GNU
D(3,1)=0.
D(1,2)=GNU
D(2,2)=1.
D(3,2)=0.
D(1,3)=0.
D(2,3)=0.
D(3,3)=.5*(1.-GNU)

C           CONVERT T MATRIX TO A VECTOR FOR INPUT TO SUBROUTINE MINV OF SSP
L=0
DO 30 J=1,16
DO 30 I=1,16
L=L+1
ALIN(L)=T(I,J)
30      INVERT MATRIX T
CALL MINV (ALIN,16,DET,LL,MM)
CHECK THAT DET=DETERMINANT IS NON-ZERO
IF (DET.NE.0) GO TO 40
WRITE (KWRITE,3020)

```



```
3020 FORMAT(68HOT MATRIX IS SINGULAR. EXECUTION IS TERMINATED IN SUBROUTINE TDGMAT./1H1)
      STOP
      CONVERT ALIN TO A 16X16 MATRIX=THE G MATRIX
C   40      L=0
          DO 50 J=1,16
          DO 50 I=1,16
          L=L+1
          G(I,J)=ALIN(L)
          RETURN
      END
      50
```


SUBROUTINE HILD (N, M, ICON, NCON, IQ, INTGR, RFLG)
 THIS SUBROUTINE GENERATES THE BOUNDARY CONDITIONS ASSUMING
 THAT ONLY THE LOWER RIGHT CORNER OF A RECTANGULAR PANEL IS SWINGING
 CONSIDERED. THE ENTIRE PANEL IS ASSUMED TO BE WELDED AROUND THE EDGES
 DIMENSION INTGR(2), RFLG(2)
 JAY=ICON-1
 JDJF1=-3
 NPL=M+1
 NP2=N+1
 DO 120 JJ=1, M+1
 DO 100 II=1, NPL
 DEFINE THE GLOBAL INDEX FOR THE AT Y(II), X(JJ)
 JDJF1=JDJF1+4
 JDJF2=JDJF1+1
 JDJF3=JDJF2+1
 JDJF4=JDJF3+1
 TEST TO SEE IF (X, Y) IS AN EDGE POINT
 IF (JJ.NE.4PL.AND.JJ.NE.1.AND.II.NE.NPL.AND.II.NE.1) GO TO 100
 IS AN EDGE POINT
 IF (JJ.EQ.1) GO TO 11
 GO TO 40
 LEFT EDGE
 JAY=JAY+1
 INTGR(JAY)=JDJF2
 JAY=JAY+1
 INTGR(JAY)=JDJF4
 IF (II.EQ.1) GO TO 20
 IF (II.EQ.NPL) GO TO 30
 GO TO 100
 LOWER LEFT CORNER
 JAY=JAY+1
 INTGR(JAY)=JDJF2
 GO TO 120
 UPPER LEFT CORNER
 JAY=JAY+1
 INTGR(JAY)=JDJF1

C 40 JAY=JAY+1
 INTG(R(JAY))=JDOF3
 JAY=JAY+1
 INTGR(JAY)=JDOF4
 GO TO 100
 ON LCESE EDOGE

C 50 JAY=JAY+1
 INTGR(JAY)=JDOF1
 JAY=JAY+1
 INTGR(JAY)=JDOF2
 GO TO 100
 ON PIGHT EDOGE

C 70 JAY=JAY+1
 INTGR(JAY)=JDOF1
 JAY=JAY+1
 INTGP(JAY)=JDOF5
 IF (II.EC.1) GO TO 30
 IF (II.FQ.JD1) GO TO 90
 GO TO 100
 ON LCEFR RING+T C PVER

C 80 JAY=JAY+1
 INTGR(JAY)=JDOF4
 GO TO 100
 ON UPPER RIGHT C PVER

C 90 JAY=JAY+1
 INTGF(JAY)=DODF2
 120 CONTINUE
 120 CONTINUE


```
NCN=JAY
DO 130 I=1, NCN
  KSUB=ICON-1+I
  NSUB=INTGP(KSUB)
  LSUB=I-1+NSUB
  REAL(LSUB)=0.
130  CONTINUE
RETURN
END
```

130


```

SUBROUTINE NDDDE(REAL, INTGR)
DIMENSION REAL(2), INTGR(2)

COMMON /BESTIV/ ICON, IKJUNT, ILNZ, IMASTR, IO, IK
COMMON /ENDV/ LCCN, LCOUNT, LLNZ, LMASTP, LJ, LK
COMMON /CONST/ A, N, A, B, CX, CY, BETA, THETAZ, ARBET, HALFA, HALFZ, ASG, ACU
1 P, ASSO, ACJB, CXR2, CYA2, RTHCXY, ATHCY
COMMON /MESH/ INDF, NNT, LTYPE(200)

THIS SUBROUTINE CALCULATES THE GLOBAL DOF NUMBERS
THAT CORRESPOND TO THE ELEMENT DOFs.

KSUB=I4ASTR-1

N4P1=4*(N+1)

KPOINT=LMASTR+NFT

DO 40 J=1, M
DO 30 I=1, N
L40M=LI+(JJ-1)*N
INTFSINAL=(NO EDGE WELDED)
IF (LI.LT.N.AND.JJ.LT.M) LTYPE(LNUM)=1
ALONG RIGHT EDGE (ONLY RIGHT EDGE WELDED)
IF (LI.NE.N.AND.JJ.EQ.N) LTYPE(LNUM)=2
ALONG TOP EDGE (ONLY TOP EDGE WELDED)
IF (LI.EQ.N.AND.JJ.LT.M) LTYPE(LNUM)=3
UPPER RIGHT CORNER (TOP EDGES WELDED)
IF (LI.EQ.N.AND.JJ.EQ.N) LTYPE(LNUM)=4
KSUB=KSUB+1
INTGR(KSUB)=<PUT
KDOF=4*(LI+N+JJ-1)-3
DO 20 KK=1, 4
LNGFR LEFT CORNER OF MESH
KK=KK-1
KSUB1=KSUB+1
INTGP(KSUB1)=KDOF+K
LNGFR RIGHT CORNER OF MESH
KSUB2=KSUB1
KSUB1=KSUB1+4
INTGP(KSUB1)=INTGP(KSUB2)+N4P1
UP2FR RIGHT CORNER OF MESH

```



```

KSUM2=KSJ31
KSUB1=KSJ31+4
INTGR(KSJ31)=INTEGER((KSUB2)+4
IF OFF LEFT CORNER OF MESH
KSUB1=KSUB1+4
KSUP2=KSJ32-4
INTGR(KSJ32)=INTEGER((KSU32)+4
CONTINUE
PUT=PUT+16
CONTINUE
CONTINUE
LAST=KSUM1
IF LMASTER.GT.LAST) GO TO 50
RETURN
KSUB1=LAST+1
DO 60 I=KSUB1,LMASTER
INTGE(I)=0
60 RETURN
END

```



```

SUBROUTINE STIFF(0,6,STIFFM,Q)
C THIS SUBROUTINE CALCULATES THE ELEMENT STIFFNESS MATRIX FOR NO WELD
C DIMENSION H(3,15),P(16,16),HTOH(16,16),STIFFM(16,16),Q(16),WF(16)
C DIMENSION D(3,3),S(15,16)
COMMON /10/ KREAO,KWHITE,KPUNCH
COMMON /CINST/ A,N,A,R,CX,CY,SETA,THETAZ,ARBET,HALFA,HALFB,ASO,ACU
13,RSO,CUR,CX32,CY12,PTHGX,ATHCY
COMMON /GAUSS/ XBAR(3),XPBAR(3),YBAR(3)
CALL ERASE(4,43,STIFFM,256,P,256,Q,16)
H(1,5)=2.
H(2,6)=2.
H(3,4)=2.
DO 48 I=1,3
X5I=X3AI(I)
JI=W3AQ(I)
X=HALFA*(X3I+1)
H(1,7)=6.*X
H(2,5)=2.*X
H(2,12)=H(2,7)*X
H(2,14)=H(2,12)*X
H(2,8)=4.*X
H(3,11)=4*(1,7)*X
47 48 J=1,3
Y2J=YBAR(J)
WJ=HALFB*(Y3J+1)
Y=HALFB*(Y3J+1)
N1=NJ=41*QJ
EVALUATE P(X,Y) AT X1,YJ
FIRST EVALUATE H AT X1,YJ
H(1,8)=2.*X
H(1,11)=H(1,7)*Y
H(1,12)=H(1,5)*Y
H(1,14)=H(1,11)*Y
H(1,15)=H(1,12)*Y
H(1,16)=H(1,14)*Y
H(2,10)=2.*Y

```



```

H(2,13)=H(1,11)
H(2,15)=H(2,13)*X
H(2,16)=H(2,15)*X
H(3, 9)=E.*Y
H(3,12)=H(2,9)*H(3,9)
H(3,13)=H(2,10)*Y
H(3,14)=H(1,8)*H(3,11)
H(3,15)=H(2,9)*H(3,13)
H(3,16)=H(1,14)*Z.*X
C COMPUTE PRODUCT OF (H TRANSPOSE) X(D) X(H) X(W) X(W)
D 40 L=1,16
D 40 J=1,16
HTOH(II,JJ)=0.
D 30 LL=1,3
DC 30 KK=1,3
HTOH(II,JJ)=HTOH(II,JJ)+H(LL,II)*H(LL,KK)*H(KK,JJ)
CONTINUE
30 HTOH(II,JJ)=H(J*HTOHL(II,JJ))
CONTINUE
40 ADD HTOH INTO TOTAL MATRIX
D 25 LI=1,15
D 25 JJ=1,15
P(LI,JJ)=P(LI,JJ)+HTOHL(II,JJ)
CONTINUE
48 COMPUTE (G TRANSPOSE) X(INTEGRAL P) X(G)
D 60 LI=1,15
D 60 JJ=1,15
D 50 LL=1,16
D 50 KK=1,16
STIFFM(II,JJ)=STIFFM(II,JJ)+S(LL,II)*P(LL,II)*G(KK,JJ)
CONTINUE
50 STIFFNESS MATRIX FOR AN ELEMENT NOT ON WELDED EDGE
52 STIFFM(II,JJ)=ABSET*STIFFM(II,JJ)
CONTINUE
53 STIFFNESS MATRIX IS COMPLETE AND ALL Q'S ARE ZERO
RETURN
END

```


SUBROUTINE STIFFX(G,STIFFX,QX)
 ADDITIONAL TERMS FOR STIFFNESS MATRIX FOR AN ELEMENT WITH WELDED
 EDGE PARALLEL TO X AXIS
 DIMENSION G(16,16),QX(16),STIFFX(16,16)
 COMMON /IOZ_KPEAO/,KARITE,KPUNCH
 COMMON /GAUSS/,WEAR(3),XRAR(3),YRAR(3)
 COMMON /CONST/V,N,A,B,CX,CY,RET,A,THCXY
 13,BSO,BCUS,CXB2,CYA2,BTHCX,A,THCXY
 COMMON /EXTRA/P(16,16),WF(16),FX(16),EKX(16,16)
 CALL FRASE(P,256,WF,16,QX,15,EKX,256)
 Y=0

FX(1)=0.

E(X(2))=0.

FX(3)=1.

FX(5)=0.

FX(6)=2.*Y

FX(7)=0.

FX(10)=3.*Y*Y

DO 150 JJ=1,3

WJ=XRAR(JJJ)

CONST=ATHCY*WJ
 $X \equiv \text{HALFA} + CXBAR(JJJ) + 1$

FX(4)=X

FX(8)=X*X

FX(11)=X*X*X

E(X(12))=FX(Q)*X

FX(13)=FX(1)*X

FX(14)=EX(12)*X

FX(15)=FX(13)*X

FX(16)=FX(15)*X

DO 120 KK=1,16

WF(KK)=WF(KK)+CONST*FX(KK)
 DO 120 LL=1,15
 $P(LL,KK)=P(LL,KK) + J*FX(LL)*FX(KK)$


```

130 CONTINUE
131 150 II=1,16
132 150 JJ=1,16
133 IX(II)=0*X(II)+G(JJ,II)*MF(JJ)
134 KK=1,15
135 DO 140 LL=1,16
136 FXX(II,JJ)=FXX(II,JJ)+S(LL,II)*P(LL,KK)*G(KK,JJ)
137 CONTINUE
138 STIFF=(II,JJ)=CYAN*EXX(II,JJ)
139 CONTINUE
140 END

```


SUBROUTINE STIFFY(G,STIFFY,QY)
 ADDITIONAL TERMS FOR STIFFNESS MATRIX FOR AN ELEMENT WITH WELD
 EDGE PARALLEL TO Y AXIS
 DIMENSION G(16,16),QY(16),STIFFY(16,16)
 COMMON /IO/KREAD,KWRITE,KPUNCH
 COMMON /GAUSS/WPAR(3),XBAR(3),YBAR(3)
 COMMON /CONST/4,N,A,B,C,X,CY,RETA,THETAZ,AR3ET,HALFA,HALFP,ASSO,ACU
 1,B,ESO,SUS3,CXA2,RTHCX,ATHCY
 COMMON /EXTRA/P(16,16),WF(16),FY(16),EKY(16,16)
 CALL ERASE(0,256,NE,16,0Y,16,0Y,256)
 X=A
 FY(1)=0.
 FY(2)=1.
 FY(3)=0.
 FY(5)=2.*X
 EY(6)=0.
 FY(7)=3.*X*X
 FY(10)=0.
 DO 8C J=1,3
 WJ=WBAR(J,J)
 CONST=RTHCX**X
 Y=HALFP*(Y3AE(J,J)+1)
 FY(4)=Y
 FY(8)=FY(5)**Y
 FY(C)=Y**Y
 FY(11)=FY(7)*Y
 FY(12)=FY(9)*Y
 FY(13)=FY(Q)*Y
 FY(14)=FY(11)*Y
 FY(15)=FY(12)*Y
 FY(16)=FY(14)*Y
 DO 70 KK=1,15
 NF(KK)=WF(KK)+CONST*FY(KK)
 DO 70 LL=L,1,
 P(LL,KK)=P(LL,KK)+EY(LL)*FY(KK)*Y
 CONTINUE


```

30      CCNTIVUC
30      DD 170 11=1,16
30      DD 170 3J=1,16
30      QY(II)=QY(II)+G(JJ,II)*WF(JJ)
30      DO   C) KK=1,15
30      DO   C) LL=1,15
30      FKY(II,JJ)=ECY(II,JJ)+G(LL,II)*P(LL,KK)*C(KK,JJ)
30      CINTTNUE
30      STIFFY(II,JJ)=CXB2*CY(II,JJ)
30      CINTTNUE
30      RETURN
30      END
100

```



```

SUBROUTINE ASEMBL(LNUM,NDE,ELK,ELQ,REAL,INTGR)
C*****ASEMBL(LNUM,NDE,ELK,ELQ,REAL,INTGR)
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C AEROELASTIC AND STRUCTURES RESEARCH LABCRATORY
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY
C*****ASEMBL(LNUM,NDE,ELK,ELQ,REAL,INTGR)
DIMENSION REAL(2),INTGR(2)
DIMENSION ELK(NDE,NDE), ELQ(NDE)
C*****ASEMBL(LNUM,NDE,ELK,ELQ,REAL,INTGR)
C CHANGE THIS DIMENSION STATEMENT IF YOU HAVE RECEIVED AN ERROR MESSAGE
C INDICATING LENGTH OF MNUM VECTOR HAS BEEN EXCEEDED. ALSO CHANGE
C INSTRUCTION INDICATED BELOW
DIMENSION MNUM(100)
C*****ASEMBL(LNUM,NDE,ELK,ELQ,REAL,INTGR)
COMMON /IO/ KR, KW, KP, KTL, KT2, KT3
COMMON /BEGIN/ ICON,IKOUNT,IINZ,IMASTR,IO,IK
C
C COMMON /DYNAM/ IKQUM1,IQM1,MNUM,INDEX,I,J,LROW,LCOL,MRW,MCOL,
2 KADR,LENGTH
C
C PRINT CCNT ROL
901 FORMAT(1HO,11O,27H DDF/ELEMENT SPECIFIED FOR ASSEMBLY NO., ASEMO023
1 MNUM VECTOR,/1X,44ERROR OCCURED DURING ASSEMBLY OF ELEMENT NO., ASEMO024
2 11O,/1X,93USER MUST CORRECT CALL ARGUMENTS IN MAIN OR CHANGE LEASEM0025
3NGTH OF MNUM VECTOR IN SUBROUTINE ASEMBL,/1X,20HEXECUTION TERMINAASEM0025
4TED)
IKQUM1 = IKOUNT-1
IQM1 = IO-1
C*****ASEMBL(LNUM,NDE,ELK,ELQ,REAL,INTGR)
C VALUE OF LENGTH MUST = DIMENSION OF MNUM
LENGTH = 100
C*****ASEMBL(LNUM,NDE,ELK,ELQ,REAL,INTGR)
IF (NDE .LE. LENGTH) GO TO 1
WRITE (KW,901) NDE, LENGTH, LNUM
STOP
C GET MASTER ROW/COL NOS FOR ELEMENT AND STORE IN MNUM

```



```

1 INDEX = IMASTR+LNUM-1
  DO 2 I = 1, NDE
    J = INTGR(INDEX)+I-1
2 MNUM(I) = INTGR(J)
C LOOP OVER ROWS OF ELQ AND OVER LOWER TRIANGLE OF ELEMENT K MATRIX
DO 4 LROW = 1, NDE
  INDEX = IQM1+MNUM(LROW)
C ASSEMBLE ELEMENT EQUIVALENT NODAL FORCE INTO Q VECTOR
  REAL(INDEX) = REAL(INDEX)+ELQ(LROW)
  DO 4 LCOL = 1, LROW
    MROW = MNUM(LROW)
    MCOL = MNUM(LCOL)
    IF (MROW .GE. MCOL) GO TO 3
    MROW = MNUM(LCOL)
    MCOL = MNUM(LROW)
C CALCULATE ABSOLUTE ADDRESS OF K(MROW,MCOL)
3 INDEX = IKNUM1+MROW
    KADR = INTGR(INDEX)+MCOL
C ASSEMBLE STIFFNESS COEFFICIENT
4 REAL(KADR) = REAL(KADR)+ELK(LROW,LCOL)
    RETURN
END

```



```

SUBROUTINE BCN(REAL,INTGR)
C***** FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C AEROELASTIC AND STRUCTURES RESEARCH LABCRATORY
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY
C***** DIMENSION REAL(2),INTGR(2)
COMMON /IO/ KR, KW, KP, KT1, KT2, KT3
COMMON /SIZE/ NET, NDT
COMMON /BEGIN/ ICUN, IKOUNT, ILNZ, LMASTR, IQ, IK
COMMON /END/ LCON, LKUUNT, LLNZ, LMASTR, LQ, LK

C COMMON /DYNAM/ LAST,IFLAG,I,J,MROW,M,NEXT,NROW,ICOL,KADR,N,
2 INIT,MCOL,ILNZMI,IKOUMI,IQMI

C PRINT CONTROL
901 FORMAT(79HODISPLACEMENT CONSTRAINTS HAVE BEEN APPLIED TO THE FJLLOBCON0017
1WING DEGREES OF FREEDOM,/ ,5X,7HDOF NO.,2X,12HDISPLACEMENT)
902 FORMAT(2X,110,2X,E10.3)
903 FORMAT(1X,72H*****ABOVE DOF NUMBER APPEARS TWICEBCON0020
1 IN CONSTRAINT LIST,/ ,1X,85HEXECUTION TERMINATED IN SUBROUTINE BCN0018
2N DUE TO POSSIBILITY OF BOUNDARY CONDITION ERROR)
904 FORMAT(62HYOUR STRUCTURE IS FLYING FREE. PLEASE CONSTRAIN IT NEXTBCON0023
1 TIME./ ,1X,28HEXECUTION TERMINATED IN BCN)
IKOUMI = IKOUNT-1
ILNZMI = ILNZ-1
IQMI = IQ-1

C PRINT ENTRY MESSAGE
WRITE (KW,901)
C ORDER THE CONSTRAINT ROW NUMBERS IN ASCENDING SEQUENCE
LAST = LCON-1
IF (LCON .GT. 0) GO TO 1
WRITE (KW,904)
STOP
1 IFLAG = 0
DO 2 I = ICUN,LAST
      BCN0001
      BCN0002
      BCN0003
      BCN0004
      BCN0005
      BCN0006
      BCN0007
      BCN0008
      BCN0009
      BCN0010
      BCN0011
      BCN0012
      BCN0013
      BCN0014
      BCN0015
      BCN0016
      BCN0017
      BCN0018
      BCN0019
      BCN0020
      BCN0021
      BCN0022
      BCN0023
      BCN0024
      BCN0025
      BCN0026
      BCN0027
      BCN0028
      BCN0029
      BCN0030
      BCN0031
      BCN0032
      BCN0033
      BCN0034
      BCN0035
      BCN0036

```



```

IF (INTGR(I+1) .GE. INTGR(I)) GO TO 2
J = INTGR(I)
INTGR(I) = INTGR(I+1)
INTGR(I+1) = J
IFLAG = 1

2 CONTINUE
  IF (IFLAG .EQ. 1) GO TO 1
  C CHECK TO SEE IF ANY ROW NUMBERS HAVE BEEN ENTERED IN CONSTRAINT
  C VECTOR - ABORT THE RUN IF NONE HAVE BEEN
  J = 0
  DO 100 I = ICON,LCUN
100  J = J+INTGR(I)
  IF (J .GT. 0) GO TO 200
  WRITE (KW,904)
  STOP
  C OUTPUT CONSTRAINT LIST
  200 DO 4 I = ICON,LCUN
    IF (INTGR(I) .EQ. 0) GO TO 4
  C CHECK FOR REPEATED DDF AFTER 1ST ONE
    IF (I .EQ. ICON) GO TO 3
    IF (INTGR(I) .NE. INTGR(I-1)) GO TO 3
    WRITE (KW,903)
    STOP
3  J = IQM1+INTGR(I)
  WRITE (KW,902) INTGR(I), REAL(J)
4 CONTINUE
  C LOOP OVER CONSTRAINED DDF FÜR MODIFICATION OF COMPLETELY ASSEMBLED
  C FORCE VECTOR
  DO 71 I = ICON,LCUN
    IF (INTGR(I) .EQ. 0) GO TO 71
  C CHECK IF PRESCRIBED DISPLACEMENT = 0 -- IF IT DOES, SKIP FORCE VECTOR
    MROW = INTGR(I)
    M = IQM1+MROW
    IF (REAL(M) .EQ. 0.) GO TO 71
  C DISPL .NE. 0 -- LOOP OVER ALL ROWS TO MODIFY FORCE VECTOR -- SKIP
  C CONSTRAINED ROWS (CONTROLLED BY VALUE OF NEXT)

```



```

NEXT = ICON
DO 7 NROW = 1, NDT
  IF (NROW .NE. INTGR(NEXT)) GO TO 5
  NEXT = NEXT+1
GO TO 7
5 IF (NROW .GT. NROW) GO TO 51
C CHECK FOR COUPLING OF ROW NROW WITH COL MROW
  J = ILNZM1+NROW
  IF (INTGR(J) .GT. MROW) GO TO 7
  IROW = NROW
  ICOL = MROW
  GO TO 6
C CHECK FOR COUPLING OF ROW MROW WITH COL NROW
51 J = ILNZM1+MROW
  IF (INTGR(J) .GT. NROW) GO TO 7
  IROW = MROW
  ICOL = NROW
C SUBTRACT K*(PRESCR DISPL) FROM FORCE VECTOR
  6 KADR = IKQUM1+IROW
  KADR = INTGR(KADR)+ICOL
  N = IQM1+NROW
  REAL(N) = REAL(N)-REAL(KADR)*REAL(M)
  7 CONTINUE
71 CONTINUE
C LOOP OVER CONSTRAINED ROWS TO DECOUPLE THEM FROM REST OF K MATRIX
DO 11 I = ICON, LCON
  IF (INTGR(I) .EQ. 0) GO TO 11
  MROW = INTGR(I)
  INIT = ILNZM1+MROW
  INIT = INTGR(INIT)
  C SET ROW = 0
  M = IKQUM1+MROW
  M = INTGR(M)
  DO 8 MCOL = INIT, MROW
    KADR = M+MCOL
    8 REAL(KADR) = 0.

```



```

C SET COLUMN = 0 IN ROWS WHOSE LNZE COL NO IS .LE. MROW -- SKIP THIS
C SECTION IF MROW IS THE LAST ROW
  IF (MROW .EQ. NDT) GO TO 10
  INIT = MROW+1
  DO 9 NROW = INIT,NDT
    N = ILNZM1+NROW
    IF (INTGR(N) .GT. MROW) GO TO 9
    KADR = IKOUM1+NROW
    KADR = INTGR(KADR)+MROW
    REAL(KADR) = 0.
 9 CONTINUE
  C SET DIAGONAL ENTRY = 1
 10 KADR = M+NROW
    REAL(KADR) = 1.
 11 CONTINUE
    RETURN
  END

```



```

SURFACE: FACTSD(REAL,INTGR)
C***** ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C AERODESIGN INC. 2002 STRUCTURES RESEARCH LABORATORY
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY
C THIS IS ALSO SUBROUTINE FACTD - SEE ENTRY POINT BELOW
      DIMENSION REAL(2),INTGR(2)
      COMMON /10/ KR, KW, KP, KT1, KT2, KT3
      COMMON /SIZE/ NDT, NDT
      COMMON /BEGIN/ ICCN,IKOUNT,ILNZ,IMASTR,IQ,IK
      COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK
      C
      COMMON /DYNAM/ NPU,I,NEXT,JRCW,MPD,TEST,MROW,M,MM,MCOL,SUM,NN,
      2 INIT,N,LAST,J,KADRM,KADRN,JJ,KADR,TESTR,IROW,IERR,IKOUNT,ILNZM1
      C
      C PRINT CONTROL
      901 FORMAT (1HO,47X,25HTRIPLE FACTOR ENTRY POINT,/,,53H K MATRIX NOT POFACTOOL8
      2SITIVE-DEFINITE IN THE FOLLOWING ROWS )
      902 FORMAT (4OX,1I2)
      903 FORMAT (43X,4HN ONE )
      904 FORMAT (58H USER SPECIFIED SEMI-DEFINITE MATRIX. EXECUTION CONTINFACTOOL22
      2UES )
      905 FORMAT (40H USER SPECIFIED POSITIVE-DEFINITE MATRIX,/,,21H EXECUTIOFACTOOL4
      2N TERMINATES )
      906 FORMAT (25HOK MATRIX SINGULAR IN ROW,1I12,/,,21H EXECUTION TERMINATEFACTOOL6
      2S )
      907 FORMAT (61HOLARGEST ROUNDING ERROR IN DIAGONAL FACTORING OCCURRED FACTOOL8
      2IN ROW,1I12,/,,43H NUMBER OF LOWEST SIGNIFICANT FIGURES LOST=,I3) FACTOOL9
      908 FORMAT (42H ROUNDING ERROR EXCEEDS ACCEPTABLE MAXIMUM,/,,21H EXECUTFACTOOL30
      2ION TERMINATES )
      C SET ENTRY FLAG
      NPD = 0
      GO TO 1
      ENTRY FACTPD(REAL,INTGR)
      NPD = 1

```



```

C PRINT ENTRY MESSAGE
 1 WRITE (KW,901)
   IKOUNT = IKOUNT-1
   ILNZM1 = ILNZ-1

C INITIALIZE AT 1ST NONZERO ENTRY IN CONSTRAINT VECTOR
DO 2 1 = IICON,LCON
  IF (INTGR(I) .EQ. 0) GO TO 2
  GO TO 3

2 CONTINUE
 3 NEXT = I
  IF (INTGR(NEXT) .EQ. 1) NEXT = NEXT+1

C INITIALIZE ERROR PARAMETERS
  JROW = 1
  MPD = 0

TEST = 1.
C DO FIRST ROW AS SPECIAL CASE
  IF (REAL(IK) .EQ. 0.) GO TO 14
  IF (REAL(IK) .GT. 0.) GO TO 4

C NPD MESSAGE AND FLAG
  WRITE (KW,902) JROW
  MPD = 1

C LOOP OVER REMAINING ROWS
 4 DO 13 MRDW = 2,NDT
    C CHECK FOR CONSTRAINED ROW - SKIP IF FOUND AND RESET FOR THE NEXT ONE
    IF (MRW .NE. INTGR(NEXT)) GO TO 5
    NEXT = NEXT+1
    GO TO 13

C FREE ROW - FACTOR FROM LNZ COL NO TO ROW NO
 5 M = ILNZM1+MRDW
  M = INTGR(M)
  MM = IKOUNT+MRDW
  MM = INTGR(MM)
  DO 12 MCOL = M,MRDW
    SUM = 0.
    NN = IKOUNT+MCOL
    NN = INTGR(NN)

```



```

C LNZ IS A SPECIAL CASE - NO SUM REQUIRED
      IF (MCOL .EQ. M) GO TO 7
C START SUM FROM GREATEST OF MRDW OR ROW *MCOL* LNZ COL NOS
      INIT = N
      N = ILNZM1+MCOL
      IF (INTGR(N) .GT. M) INIT = INTGR(N)
      C NO SUM IF ROW *MCOL* HAS LEADING ZEROS UP TO THE DIAGONAL
      IF (INIT .EQ. MCOL) GO TO 7
      C ACCUMULATE THE SUM
      LAST = MCOL-1
      DO 6 J = INIT, LAST
      KADRM = MM+J
      KADRN = NN+J
      JJ = IKOUM1+J
      JJ = INTGR(JJ)+J
      6 SUM = SUM+REAL(JJ)*REAL(KADRM)*REAL(KADRN)
      C BRANCH TO SPECIAL ALGORITHM FOR DIAGONAL ENTRIES
      7 IF (MCOL .EQ. MRDW) GO TO 8
      C FOR OFF-DIAGONAL ENTRIES:
      KADR = MM+MCOL
      NN = NN+MCOL
      REAL(KADR) = (REAL(KADR)-SUM)/REAL(NN)
      GO TO 12
      C DIAGONAL ENTRY - TEST FOR SINGULARITY AND SEMI-DEFINITNESS
      8 MM = MM+MRDW
      IF (REAL(MM)-SUM .NE. 0.) GO TO 9
      JRDW = MRDW
      GO TO 14
      9 IF (REAL(MM)-SUM .GT. 0.) GO TO 10
      WRITE (KW, 902) MRDW
      MPD = 1
      C CALCULATE ROUNDING ERROR
      10 TESTR = ABS((REAL(MM)-SUM)/REAL(MM))
      IF (TESTR .GE. TEST) GO TO 11
      TEST = TESTR
      IRDW = MRDW
      FACT0073
      FACT0074
      FACT0075
      FACT0076
      FACT0077
      FACT0078
      FACT0079
      FACT0080
      FACT0081
      FACT0082
      FACT0083
      FACT0084
      FACT0085
      FACT0086
      FACT0087
      FACT0088
      FACT0089
      FACT0090
      FACT0091
      FACT0092
      FACT0093
      FACT0094
      FACT0095
      FACT0096
      FACT0097
      FACT0098
      FACT0099
      FACT0100
      FACT0101
      FACT0102
      FACT0103
      FACT0104
      FACT0105
      FACT0106
      FACT0107
      FACT0108

```



```

C EVALUATE DIAGONAL ENTRY
11 REAL(MM) = REAL(MM)-SUM
12 CONTINUE
13 CONTINUE
C SEMI-DEFINITENESS CHECKS AND ROUNDING ERROR OUTPUT
    IF (MPD .EQ. 0) WRITE (KW,903)
    IF (MPD .EQ. 1 .AND. NPD .EQ. 1) WRITE (KW,905)
    IF (MPD .EQ. 1 .AND. NPD .EQ. 0) WRITE (KW,904)
    IERR = -1.00001*ALOG10(TEST)
    WRITE (KW,907) IROW,IERR
    IF (IERR .GT. 5) WRITE (KW,908)
    IF (MPD .EQ. 1 .AND. NPD .EQ. 1) STOP
    IF (IERR .GT. 5) STOP
    RETURN
C SINGULAR MATRIX
14 WRITE (KW,906) JROW
    STOP
END
FACT0109
FACT0110
FACT0111
FACT0112
FACT0113
FACT0114
FACT0115
FACT0116
FACT0117
FACT0118
FACT0119
FACT0120
FACT0121
FACT0122
FACT0123
FACT0124
FACT0125
FACT0126

```



```

SUBROUTINE ORK( LENGTH, REAL, INTGR )
C***** THIS SUBROUTINE CREATES
C   1) THE LNZ VECTOR WHICH HOLDS THE COLUMN NUMBER
C      OF THE LEADING NON-ZERO ENTRY IN EACH ROW OF
C      THE ASSEMBLED "K" MATRIX
C   2) THE ADDRESS COUNT VECTOR WHICH HOLDS THE ABSOLUTE
C      ADDRESS OF THE DIAGONAL ENTRY FOR EACH ROW OF THE
C      ASSEMBLED K MATRIX, MINUS THE ROW NUMBER, AND
C      CHECKS THAT K FITS IN THE /DATA/ VECTOR
C
C DIMENSION REAL(2),INTGR(2)
COMMON /10/ KR, KW, KP, KT1, KT2, KT3
COMMON /SIZE/ NET, NDT
COMMON /BECIN/ ICON, IKOUNT, ILNZ, IMASTR, IQ, IK.
COMMON /END/ LCJN, LKOUNT, LLNZ, LMASTR, LQ, LK
C
COMMON /DYNAM/ ILNZM1, IMSTM1, IKOUM1, NETM1, IMSTP1, IROW, MSUB, LNUM,
2 MADDR, NDE, ISMALL, JDOF, INDEX, NENTRY, DENS, I, J
C
C PRINT CONTROL
100 FORMAT(49HOTHE LENGTH OF THE "DATA" VECTOR FOR THIS CASE IS,1I0,14I0
1H WHICH EXCEEDS,1I0,49H =THE MAXIMUM ALLOWED IN THE DIMENSION STATORK
2EMENT. /39H EXECUTION TERMINATED IN SUBROUTINE ORK)
200 FORMAT(5X,3HROW,2X,13HLNE COL. NO.,2X,18HABS. ADR. OF DIAG.)
300 FORMAT(19,1I0,1I2)
400 FORMAT(10HOTHER ARE,1I0,68H NON-ZERO ENTRIES.,,20X,15HTHE DENSIORK
1FULLY POPULATED THERE WOULD BE,1I0, 9H ENTRIES.,,20X,15HTHE DENSIORK
2TY 1S ,E15.6)
ILNZM1=ILNZ-1
IMSTM1=IMASTR-1
IKOUM1=IKOUNT-1
NETM1=NET-1

```



```

C SET EACH LNZ COLUMN NO = ROW NO (DIAGONAL MATRIX)
DO 30 IROW=1,NDT
  MSUB=ILNZM1+IRCW
  30  INTGR(MSUB)=IROW
C EXAMINE 'MASTER ASSEMBLY LIST', ONE ELEMENT AT A TIME, TO CREATE
C THE LNZ VECTOR
  DO 20 LNUM=1,NET
    MADDR = IMSTM1+LNUM
    MADDR = INTGR(MADDR)-1
C CALCULATE NO. DOF IN THE ELEMENT BY DIFFERENCING POINTERS, OR ...
    I = IMASTR+LNUM
    IF(LNUM.EQ.NET) GO TO 3
    NDE = INTGR(I)-INTGR(I-1)
    GO TO 4
C ... BEGIN BY ASSUMING THE LIST IS FILLED, FOR LAST ELEMENT
    3  NDE = LMASTR-INTGR(I-1)+1
C INITIALIZE SMALLEST DOF NO. AT LARGEST POSSIBLE VALUE
    4  ISMALL=NDT
C FIND SMALLEST MASTR NUMBER FOR THIS ELEMENT
    DO 5 JDOF=1,NDE
      INDEX=MADDR+JDOF
      IF(INTGR(INDEX).GT.ISMALL) GO TO 5
C DISCONTINUE SEARCH IF A ZERO IS FOUND, INDICATING EXCESS STORAGE
C AND PREMATURE END OF LIST FOR LAST ELEMENT
      IF(INTGR(INDEX).EQ.0) GO TO 6
      ISMALL=INTGR(INDEX)
      CONTINUE
C FIND COLUMN NUMBER OF LEADING NON-ZERO ENTRY IN ROW
      6  DO 10 JDOF=1,NDE
        INDEX=MADDR+JDOF
        INDEX=ILNZM1+INTGR(INDEX)
        CHANGE LNZ COL NO ONLY IF NEW ONE IS LESS THAN OLD ONE
        IF(INTGR(INDEX).LT.ISMALL) GO TO 8
        INTGR(INDEX)=ISMALL
        GO TO 10

```



```

C DISCONTINUE OPERATION IF EXCESS STORAGE IS DISCOVERED
 8 IF(INTGR(INDEX).EQ.0) GO TO 20
10 CONTINUE
20 CONTINUE
C CREATE ADDRESS COUNT VECTOR
  INDEX=IKOUNT
  DO 40 IROW=2,NDT
    I = ILNZM1+IROW
    INTGR(INDEX+1) = INTGR(INDEX)+IROW+1-INTGR(I)
40 INDEX=INDEX+1
C ADDRESS COUNT VECTOR NOW CONTAINS ABSOLUTE ADDRESS ONLY FOR THE
C DIAGONAL ENTRIES, AND THUS INTGR(LKOUNT) = LK EXACTLY
C IF (INTGR(LKOUNT) • LE. LENGTH) GO TO 50
  WRITE (KW,100) INTGR(LKOUNT), LENGTH
  STOP
50 LK = INTGR(LKOUNT)
  WRITE(KW,200)
  DO 60 IROW=1,NDT
    I = ILNZM1+IROW
    J = IKOUNT1+IROW
    WRITE (KW,300) IROW, INTGR(I), INTGR(J)
    REPLACE THE ABS. ADDRESS OF DIAG. BY (ABS. ADDRESS - ROW NO.)
    INTGR(J) = INTGR(J)-IROW
    CONTINUE
    NENTRY = INTGR(LKOUNT)+NDT-IK+1
    INDEX = (NDT*(NDT+1))/2
    DENS = FLOAT(NENTRY)/FLOAT(INDEX)
    WRITE (KW,400) NENTRY, INDEX, DENS
C ZERO THE FORCE/DISPLACEMENT VECTOR AND THE K MATRIX BLOCK
    DO 70 I=IQ,LK
      REAL(I)=0.
    RETURN
  END

```



```

SUBROUTINE SETUP(LENGTH,NCON,MASTRL,REAL,INTGR)
C***** ****
C FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C AEROELASTIC AND STRUCTURES RESEARCH LABORATORY
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY
C***** ****
C
      DIMENSION REAL(2), INTGR(2)
      DIMENSION LI(6), LI(6), KD(6)
      COMMON /IO/ KR, KW, KP, KT1, KT2, KT3
      COMMON /SIZE/ NET, NDT
      COMMON /BEGIN/ ICON, IKOUNT, ILNZ, IMASTR, IQ, IK
      COMMON /END/ LCON, LKOUNT, LLNZ, LMASTR, LQ, LK
      COMMON /DYNAM/ I, TR, DENS
      EQUIVALENCE (ICON, II(1)), (LCON, LI(1)), (KR, KD(1))
      C
      C PRINT CONTROL
      901 FORMAT (1H0,53X,11HSETUP ENTRY,/,*42X,35HUSER SPECIFICATIONS TO FEAS
      1BL SYSTEM,/,*1H0,51X,16H1/O DEVICE CODES,/,*27X,6HREADER,*X,7HPRINTESETU0018
      2R,2X,10HCARD PUNCH,7X,5HTAPE1,7X,5HTAPE2,7X,5HTAPE3,/,*21X,6(2X,110SE
      3),/,*1H0,53X,12HPROBLEM SIZE,/,*42X,25HTOTAL NUMBER OF ELEMENTS=,110SE
      4,/,*42X,25HTOTAL DEGREES OF FREEDOM=,110,/,*1H0,51X,
      5ERS,/,*39X,32HASSUMED LENGTH OF /DATA/ VECTOR=,110,/,*32X,45HNUMBER SETU0023
      60F DISPLACEMENT CONSTRAINTS REQUESTED=,110,/,*33X,44HNUMBER OF WORDSETU0024
      7S REQUESTED FOR ASSEMBLY LIST=,110)                                SETU0025
      902 FORMAT (1H0,43X,31H/ DATA/ VECTOR ADDRESS INDEX MAP,/,*22X,11HCONSTR SETU0026
      1AINTS,2X,10HDG ABS ADR,1X,11HLNZE COL NO,1X,11HASMBLY LIST,2X,10HQSETU0027
      2/U VECTOR,4X,8HK MATRIX,/,*16X,5HBEGIN,6(2X,110),/,*18X,3HEND,5(2X,1SETU0028
      310),11X,1H+,/,*1H0,36H+ LK IS CALCULATED BY SUBROUTINE CRK)
      903 FORMAT (4OHOSTORAGE EXCEEDS LENGTH OF /DATA/ VECTOR,/,*1X,*34H LENGTH SETU0030
      1 SUGGESTED FOR THIS PROBLEM=,112,6H WORDS,/,*1X,*40H EXECUTION TERMINSETU0031
      2ATED IN SUBROUTINE SETUP)                                              SETU0032
      C***** ****
      C REMOVE THIS FORMAT AND WRITE STATEMENT INDICATED BELOW IF FEABL
      C HEADING IS NOT DESIRED
      1001 FORMAT(1H/3(1H,*92(1H))/,4(5H XXXX,84X,4HXXXXX/),5H XXXX,7X,1O*1HSETU0033
      SETU0034
      SETU0035
      SETU0036
      
```



```

1F), 5X, 10(1HE), 9X, 2HAA, 9X, 8(1HB), 7X, 3HLLL, 14X, 4HXXXXX/5H XXXXX, 7X, 10( SETU0037
21HF), 5X, 10(1HE), 8X, 4HAAAA, 8X, 9(1HB), 6X, 3HLLL, 14X, 4HXXXXX/5H XXXXX, 7X SETU0038
3, 3HFFF, 12X, 3HEEE, 14X, 6(1HA), 7X, 3HBBB, 3X, 4HBBBB, 5X, 3HLLL, 14X, 4HXXXXX/5H SETU0039
4/5H XXXX, 7X, 3HFFF, 12X, 3HEEE, 13X, 3HAAA, 2X, 3HAAA, 6X, 3HBBB, 4X, 3HBBB, 5SETU0040
5X, 3HLLL, 14X, 4HXXX/5H XXXX, 7X, 3HFF, 12X, 3HEEE, 12X, 3HAAA, 4X, 3HAAA, 5SETU0041
6X, 3HBBB, 4X, 3HBBB, 5X, 3HLLL, 14X, 4HXXX/5H XXXX, 7X, 3HFFF, 12X, 3HEEE, 12 SETU0042
7X, 3HAAA, 4X, 3HAA, 5X, 3HBBB, 3X, 4HBBBB, 5X, 3HLLL, 14X, 4HXXX/5H XXXX, 7X SETU0043
8, 8(1HF), 7X, 3(1HE), 7X, 3HAAA, 4X, 3HAA, 5X, 9(1HB), 6X, 3HLLL, 14X, 4HXXXXX/5H SETU0044
95H XXXX, 7X, 8(1HF), 7X, 8(1HE), 7X, 10(1HA), 5X, 9(1HB), 6X, 3HLLL, 14X, 4HXXX/5H SETU0045
AXX/5H XXXX, 7X, 3HFFF, 12X, 3HEEE, 12X, 10(1HA), 5X, 3HBBB, 3X, 4HBBBB, 5X, 3HSETU0046
BL, 14X, 4HXXX/5H XXXX, 7X, 3HFFF, 12X, 3HEEE, 12X, 3HAAA, 4X, 3HAAA, 5X, 3HSETU0047
CBBB, 4X, 3HBBR, 5X, 3HLLL, 14X, 4HXXX/5H XXXX, 7X, 3HFFF, 12X, 3HEEE, 12X, 3HSETU0048
DAAA, 4X, 3HAA, 5X, 3HBBB, 3X, 4HBBBB, 5X, 3HLLL, 14X, 4HXXX/5H XXXX, 7X, 3HFSSETU0049
EFF, 12X, 10(1HE), 5X, 3HAAA, 4X, 3HAA, 5X, 9(1HB), 6X, 10(1HL), 7X, 4HXXX/5H SETU0050
F XXXX, 7X, 3HFFF, 12X, 10(1HE), 5X, 3HAA, 4X, 3HAA, 5X, 8(1HB), 7X, 10(1HL), SETU0051
G7X, 4HXXX/4(5H XXXX, 84X, 4HXXX/1), 5H XXXX, 24X, 37HFINITE ELEMENT ANASETU0052
HYYSIS BASIC LIBRARY, 23X, 4HXXX/4(5H XXXX, 84X, 4HXXX/1), 3(1H, 92(1HXSETU0053
1), 1H1) ) **** SETU0054
C **** SETU0055
C PRINT ENTRY MESSAGE **** SETU0056
C **** REMOVE THIS WRITE STATEMENT IF FEABL HEADING IS NOT DESIRED **** SETU0057
C **** WRITE ( KW, 1001 ) **** SETU0058
C **** WRITE ( KW, 901 ) (KD(I), I = 1,6), NET, NDT, LENGTH, NCQN, MASTRL **** SETU0059
C CALCULATE ADDRESS INDEX VALUES **** SETU0060
ICON = 1
LCQN = NCQN
IKOUNT = LCQN+1
LKCOUNT = LCOUNT+NDT
LLNZ = LCOUNT+NDT
IMASTR = LLNZ+1
LMASTR = LMASTR+NDT
IQ = LMASTR+1
LQ = LMASTR+NDT
SETU0061
SETU0062
SETU0063
SETU0064
SETU0065
SETU0066
SETU0067
SETU0068
SETU0069
SETU0070
SETU0071
SETU0072

```



```

IK = LQ+1
C PRINT INDEX MAP
WRITE (KW,902) (II(I), I = 1,6), (LI(I), I = 1,5)
C STORAGE BOUNDS TEST
IF (LMASTR .LE. LENGTH) GO TO 1
C STORAGE EXCEEDED - ESTIMATE REQUIRED LENGTH BASED ON LOWER TRIANGLE
C ESTIMATED POPULATION FACTOR
TR = (NDT*(NDT+1))/2
DENS = 0.5
IF (NDT .GT. 200) DENS=0.3
IF (NDT .GT. 500) DENS=0.2
IF (NDT .GT. 1500) DENS = 0.15
IF (NDT .GT. 2000) DENS = 0.10
TR = TR*DENS
LENGTH = LO+TR
WRITE (KW,903) LENGTH
STOP
C ENOUGH STORAGE EXISTS TO GO THRU ORK
1 DO 2 I = ICQN,LQN
2 INTGR(I) = 0
RETURN
END
SETU0073
SETU0074
SETU0075
SETU0076
SETU0077
SETU0078
SETU0079
SETU0080
SETU0081
SETU0082
SETU0083
SETU0084
SETU0085
SETU0086
SETU0087
SETU0088
SETU0089
SETU0090
SETU0091
SETU0092
SETU0093
SETU0094

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SUBROUTINE SIMULQ(ENERGY,REAL,INTGR)
C***** FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C AEROELASTIC AND STRUCTURES RESEARCH LABCRATORY
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY
C***** ****
C DIMENSION REAL (2),INTGR (2)
COMMON /IO/ KR, KW, KP, KT1, KT2, KT3
COMMON /SIZE/ NET, NDT
COMMON /BEGIN/ ICUN, IKOUNT, ILNZ, IMASTR, IQ, IK
COMMON /END/ LCUN, LKOUNT, LLNZ, LMASTR, LQ, LK
C
COMMON /DYNAM/ IKOUNI, ILNZM1, IQM1, IFLAG, MOST, LEFT, JBEG, NBEG, I,
2 JEND, NEND, J,N, NEXT, MROW, SUM, LAST, M, MM, KADR, INIT
C
C PRINT CGNTROL
901 FORMAT (1HO,53X,18HSIMULQ ENTRY POINT,/,>38H PRESCRIBED FORCE/DISPLS
2ACEMENT VECTOR: )
902 FORMAT (30HODISPLACEMENT SOLUTION VECTOR: )
903 FORMAT (1HO,2X,3HROW,10I12)
904 FORMAT (6H VALUE,10(2X,E10.3))
905 FORMAT (32HOSTRAIN ENERGY IN THE STRUCTURE=,E10.3)
IKOUNI = IKOUNT-1
ILNZM1 = ILNZ-1
IQM1 = IQ-1
C PRINT ENTRY MESSAGE, SET PRINT SECTION CONTROL FLAG
WRITE (KW,901)
IFLAG = 0
C PRINT SECTION: DIVIDE Q VECTOR INTO GROUPS OF TEN
C PLUS A POSSIBLE REMAINDER
MOST = NDT/10
LEFT = NDT-10*MOST
JBEG = 1
NBEG = IQ
C RE-ENTRY POINT FOR SOLUTION OUTPUT
1 IF (NDT .LT. 10) GO TO 3

```



```

DO 2 I = 1,MOST
JBEG = 1+10*(I-1)
JEND = JBEG+9
NREG = IQ+JBEG-1
NEND = NBEG+9
WRITE (KW,903) (J, J = JBEG,JEND)
2 WRITE (KW,904) (REAL(N), N = NBEG,NEND)
C CHECK FOR EXISTENCE OF REMAINDER
IF (LEFT .EQ. 0) GO TO 4
JREG = JEND+1
NBEG = NEND+1
3 WRITE (KW,903) (J, J = JBEG,NDT)
WRITE (KW,904) (REAL(N), N = NBEG,LQ)
C CHECK CONTROL FLAG
4 IF (IFLAG .EQ. 0) GO TO 5
WRITE (KW,905) ENERGY
RETURN
C FORWARD SOLUTION - NO DIVISIONS, SO SKIP FIRST ROW ENTIRELY
C INITIALIZE CONSTRAINT VECTOR AT 1ST NCNZROW ENTRY
5 DO 6 I = ICON,LCON
IF (INTGR(I) .EQ. 0) GO TO 6
GO TO 7
6 CONTINUE
7 NEXT = I
IF (INTGR(NEXT) .EQ. 1) NEXT = NEXT+1
C SOLVE (A)(R) = (Q)
DO 10 MROW = 2,NDT
C CHECK FOR CONSTRAINT TO SKIP ROW
IF (MROW .NE. INTGR(NEXT)) GO TO 8
C UPDATE NEXT
NEXT = NEXT+1
GO TO 10
C INITIALIZE SUM AND LOOP LIMITS FOR SUM
8 SUM = 0
LAST = MROW-1
M = ILNZ+LAST
SIMU0037
SIMU0038
SIMU0039
SIMU0040
SIMU0041
SIMU0042
SIMU0043
SIMU0044
SIMU0045
SIMU0046
SIMU0047
SIMU0048
SIMU0049
SIMU0050
SIMU0051
SIMU0052
SIMU0053
SIMU0054
SIMU0055
SIMU0056
SIMU0057
SIMU0058
SIMU0059
SIMU0060
SIMU0061
SIMU0062
SIMU0063
SIMU0064
SIMU0065
SIMU0066
SIMU0067
SIMU0068
SIMU0069
SIMU0070
SIMU0071
SIMU0072

```



```

M = INTGR(M)
MM = IKJUNT+LAST
MM = INTGR(MM)
DC 9 J = M+LAST
KADR = MM+J
N = IQM1+J
      9 SUM = SUM+REAL(KADR)*REAL(N)
C SUBTRACT SUM FROM Q
      N = IQM1+MRROW
      REAL(N) = REAL(N)-SUM
      10 CONTINUE
C SOLVE (D)(P) = (R) AND CALCULATE ENERGY
      N = IQ-1
      ENERGY = 0.
      DO 11 MRROW = 1,NDT
      KADR = IKOUM1+MRROW
      KADR = INTGR(KADR)+MRROW
      N = N+1
      REAL(N) = REAL(N)/REAL(KADR)
      11 ENERGY = ENERGY+REAL(KADR)*(REAL(N)**2)
      ENERGY = 0.5*ENERGY
C BACK SOLUTION - NO DIVISIONS, SO SKIP LAST ROW ENTIRELY
      NEXT = LCON
      IF (INTGR(NEXT) .EQ. NDT) NEXT = NEXT-1
      C LOOP BACKWARDS OVER REMAINING ROWS
      DO 14 I = 2,NDT
      MRROW = NDT+I-1
      C CHECK FOR CONSTRAINT TO SKIP ROW
      IF (MRROW .NE. INTGR(NEXT)) GO TO 12
      C UPDATE NEXT
      NEXT = NEXT-1
      IF (NEXT .LT. ICON) NEXT = LCON
      GO TO 14
C INITIALIZE SUM AND LOWER LOOP LIMIT
      12 SUM = 0.
      INIT = MRROW+1

```



```

D0 13 J = INIT,NDT
C CHECK IF LNZ COL NO OF ROW J EXCEEDS MROW - IF SO, SKIP
N = ILNZM1+J
IF (INTGR(N) • CT. MROW) GO TO 13
KADR = IKOUM1+J
KADR = INTGR(KADR)+MROW
N = IQM1+J
SUM = SUM+REAL(KADR)*REAL(N)
13 CONTINUE
C SUBTRACT SUM FROM Q
N = IQM1+MROW
REAL(N) = REAL(N)-SUM
14 CONTINUE
C PRINT SOLUTION HEADING, RESET CONTROL FLAG AND
C BRANCH TO OUTPUT SECTION
WRITE (KW,902)
IFLAG = 1
GO TO 1
END
SIMU0109
SIMU0110
SIMU0111
SIMU0112
SIMU0113
SIMU0114
SIMU0115
SIMU0116
SIMU0117
SIMU0118
SIMU0119
SIMU0120
SIMU0121
SIMU0122
SIMU0123
SIMU0124
SIMU0125
SIMU0126
SIMU0127

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```

SUBROUTINE XTRACT(LNUM,NDE,ELQ,REAL,INTGR)
C***** FINITE ELEMENT ANALYSIS BASIC LIBRARY SUBROUTINE-VERSION 2
C AEROELASTIC AND STRUCTURES RESEARCH LABORATORY
C MASSACHUSETTS INSTITUTE OF TECHNOLOGY
C***** XTRA0001 ***** XTRA0002 ***** XTRA0003 ***** XTRA0004 ***** XTRA0005 ***** XTRA0006 ***** XTRA0007 ***** XTRA0008 ***** XTRA0009 ***** XTRA0010 ***** XTRA0011 ***** XTRA0012 ***** XTRA0013 ***** XTRA0014 ***** XTRA0015 ***** XTRA0016 ***** XTRA0017 ***** XTRA0018 ***** XTRA0019 ***** XTRA0020 ***** XTRA0021 ***** XTRA0022 ***** XTRA0023

DIMENSION REAL(2),INTGR(2)
DIMENSION ELQ(NDE)
COMMON /BEGIN/ ICON,IKOUNT,ILNZ,IMASTR,IQ,IK
COMMON /END/ LCON,LKOUNT,LLNZ,LMASTR,LQ,LK

C COMMON /DYNAM/ ISTART,I,J

C FIND STARTING LOCATION OF ELEMENT ASSEMBLY LIST, -1
  ISTART = IMASTR+LNUM-1
  ISTART = INTGR(ISTART)-1
C EXTRACT DISPLACEMENTS FROM Q AND STORE IN ELQ
  DO 1 I = 1,NDE
    J = ISTART+I
    J = IQ+INTGR(J)-1
    1 ELQ(I) = REAL(J)
    RETURN
  END

```


APPENDIX E

Sample Output

DISTORTION ANALYSIS OF WELDED PANEL STRUCTURE....BY D. SHIN
 LX= 0.16000E 02, LY= 0.10000E 02, M=NO. ELEMENTS X DIR= 8
 NO. ELEMENTS Y DIR= 5, CX= 0.11300E 05, CY= 0.11300E 05
 THETA 0= 0.45000F-01 RADIANS, POISONS RATIO= 0.30000F 00
 THICKNESS= 0.37500E 00, YOUNGS MODULUS= 0.30000E 08

NODE	W	$\frac{\partial W}{\partial X}$	$\frac{\partial W}{\partial Y}$	$\frac{\partial^2 W}{\partial X^2}$
1	-0.91928E-01	0.0	0.0	0.0
2	-0.88355E-01	0.0	0.35772E-02	0.0
3	-0.77587E-01	0.0	0.72081E-02	0.0
4	-0.59449E-01	0.0	0.10954E-01	0.0
5	-0.33680E-01	0.0	0.14841E-01	0.0
6	0.0	0.0	0.18860E-01	0.0
7	-0.91103E-01	0.83277E-03	0.0	0.0
8	-0.87579E-01	0.78365E-03	0.35294E-02	-0.53047E-04
9	-0.76942E-01	0.65235E-03	0.71237E-02	-0.90185E-04
10	-0.58993E-01	0.45744E-03	0.10845E-01	-0.11218E-03
11	-0.33455E-01	0.22819E-03	0.14724E-01	-0.11745E-03
12	0.0	0.0	0.18753E-01	-0.10866E-03
13	-0.88539E-01	0.17606E-02	0.0	0.0
14	-0.85157E-01	0.16631E-02	0.33897E-02	-0.10264E-03
15	-0.74923E-01	0.13905E-02	0.68673E-02	-0.18373E-03
16	-0.57579E-01	0.97915E-03	0.10509E-01	-0.23550E-03
17	-0.32746E-01	0.49051E-03	0.14360E-01	-0.25204E-03
18	0.0	0.0	0.18416E-01	-0.23381E-03
19	-0.83915E-01	0.29142E-02	0.0	0.0
20	-0.80781E-01	0.27593E-02	0.31448E-02	-0.15960E-03
21	-0.71257E-01	0.23174E-02	0.64100E-02	-0.29382E-03
22	-0.54900E-01	0.16415E-02	0.99024E-02	-0.38679E-03
23	-0.31445E-01	0.82650E-03	0.13695E-01	-0.42260E-03
24	0.0	0.0	0.17798E-01	-0.39422E-03
25	-0.76646E-01	0.44285E-02	0.0	0.0
26	-0.73887E-01	0.42048E-02	0.27736E-02	-0.22747E-03
27	-0.65447E-01	0.35564E-02	0.57058E-02	-0.42997E-03
28	-0.50856E-01	0.25432E-02	0.89496E-02	-0.58395E-03
29	-0.29355E-01	0.12916E-02	0.12632E-01	-0.55582E-03
30	0.0	0.0	0.16801E-01	-0.61803E-03
31	-0.65859E-01	0.64462E-02	0.0	0.0
32	-0.63630E-01	0.61465E-02	0.22550E-02	-0.30303E-03
33	-0.56724E-01	0.52594E-02	0.46998E-02	-0.59186E-03
34	-0.44567E-01	0.38252E-02	0.75426E-02	-0.84170E-03
35	-0.26136E-01	0.19748E-02	0.11009E-01	-0.99117E-03

36	0.0	0.0	0.15257E-01	-0.95434E-03
37	-0.50438E-01	0.91026E-02	0.0	0.0
38	-0.48869E-01	0.87341E-02	0.15847E-02	-0.37215E-03
39	-0.43977E-01	0.76144E-02	0.33565E-02	-0.75791E-03
40	-0.35158E-01	0.57086E-02	0.55600E-02	-0.11564E-02
41	-0.21198E-01	0.30509E-02	0.85710E-02	-0.14943E-02
42	0.0	0.0	0.12358E-01	-0.15218E-02
43	-0.28973E-01	0.12489E-01	0.0	0.0
44	-0.28182E-01	0.12086E-01	0.80030E-03	-0.40724E-03
45	-0.25692E-01	0.10828E-01	0.17212E-02	-0.86705E-03
46	-0.21094E-01	0.85499E-02	0.29518E-02	-0.14512E-02
47	-0.13388E-01	0.49515E-02	0.49353E-02	-0.22364E-02
48	0.0	0.0	0.88895E-02	-0.27871E-02
49	0.0	0.16597E-01	0.0	0.0
50	0.0	0.16219E-01	0.0	-0.38120E-03
51	0.0	0.15027E-01	0.0	-0.82971E-03
52	0.0	0.12794E-01	0.0	-0.14703E-02
53	0.0	0.88826E-02	0.0	-0.27754E-02
54	0.0	0.0	0.0	-0.85932E-02

DISTORTION ANALYSIS OF WELDED PANEL STRUCTURE....BY D. SHIN
 LX= 0.16000E 02, LY= 0.10000E 02, M=NO. ELEMENTS X DIR= 8
 NO. ELEMENTS Y DIR= 5, CX= 0.42000E 05, CY= 0.42000E 05
 THETA 0= 0.55000E-01 RADIANS, POISSONS' RATIO= 0.30000E 00
 THICKNESS= 0.37500E 00, YOUNGS MODULUS= 0.30000E 08

NODE	W	$\frac{\partial W}{\partial X}$	$\frac{\partial W}{\partial Y}$	$\frac{\partial^2 W}{\partial X^2}$
1	-0.20630E 00	0.0	0.0	0.0
2	-0.19311E 00	0.0	0.81999E-02	0.0
3	-0.17348E 00	0.0	0.16446E-01	0.0
4	-0.13228E 00	0.0	0.24772E-01	0.0
5	-0.74396E-01	0.0	0.33102E-01	0.0
6	0.0	0.0	0.41245E-01	0.0
7	-0.20479E 00	0.15373E-02	0.0	0.0
8	-0.19571E 00	0.14253E-02	0.80926E-02	-0.12003E-03
9	-0.17237E 00	0.11358E-02	0.16265E-01	-0.19459E-03
10	-0.13156E 00	0.73612E-03	0.24551E-01	-0.21924E-03
11	-0.74079E-01	0.32457E-03	0.32912E-01	-0.19216E-03
12	0.0	0.0	0.41119E-01	-0.13083E-03
13	-0.19997E 00	0.33650E-02	0.0	0.0
14	-0.19222E 00	0.31367E-02	0.77720E-02	-0.23849E-03
15	-0.15878E 00	0.25173E-02	0.15705E-01	-0.40788E-03
16	-0.12922E 00	0.16449E-02	0.23897E-01	-0.47647E-03
17	-0.73044E-01	0.73174E-03	0.32306E-01	-0.43069E-03
18	0.0	0.0	0.40708E-01	-0.29460E-03
19	-0.19090E 00	0.58546E-02	0.0	0.0
20	-0.18374E 00	0.54795E-02	0.71915E-02	-0.38374E-03
21	-0.16195E 00	0.44364E-02	0.14669E-01	-0.67925E-03
22	-0.12473E 00	0.29342E-02	0.22639E-01	-0.82320E-03
23	-0.71033E-01	0.13205E-02	0.31135E-01	-0.76912E-03
24	0.0	0.0	0.39906E-01	-0.53212E-03
25	-0.17587E 00	0.93857E-02	0.0	0.0
26	-0.16963E 00	0.88291E-02	0.62826E-02	-0.56339E-03
27	-0.15045E 00	0.72449E-02	0.13005E-01	-0.10340E-02
28	-0.11705E 00	0.48832E-02	0.20552E-01	-0.13141E-02
29	-0.67547E-01	0.22379E-02	0.29125E-01	-0.12862E-02
30	0.0	0.0	0.38504E-01	-0.90881E-03
31	-0.15242E 00	0.14326E-01	0.0	0.0
32	-0.14748E 00	0.13576E-01	0.49987E-02	-0.75691E-03
33	-0.13209E 00	0.11375E-01	0.10551E-01	-0.14575E-02
34	-0.10449E 00	0.79106E-02	0.17302E-01	-0.19902E-02
35	-0.61697E-01	0.37420E-02	0.25812E-01	-0.21083E-02

36	0.0	0.0	0.36123E-01	-0.15539E-02
37	-0.11745E 00	0.20934E-01	0.0	0.0
38	-0.11416E 00	0.20047E-01	0.33369E-02	-0.89815E-03
39	-0.10373E 00	0.17337E-01	0.72541E-02	-0.18427E-02
40	-0.84243E-01	0.12707E-01	0.12542E-01	-0.28108E-02
41	-0.51826E-01	0.63954E-02	0.20411E-01	-0.34666E-02
42	0.0	0.0	0.31996E-01	-0.28367E-02
43	-0.67591E-01	0.29178E-01	0.0	0.0
44	-0.66083E-01	0.28315E-01	0.15351E-02	-0.87708E-03
45	-0.61216E-01	0.25567E-01	0.34282E-02	-0.19292E-02
46	-0.51732E-01	0.20391E-01	0.62956E-02	-0.34149E-02
47	-0.34341E-01	0.11729E-01	0.11718E-01	-0.56074E-02
48	0.0	0.0	0.24053E-01	-0.64475E-02
49	0.0	0.38519E-01	0.0	0.0
50	0.0	0.37908E-01	0.0	-0.62151E-03
51	0.0	0.35921E-01	0.0	-0.14222E-02
52	0.0	0.31951E-01	0.0	-0.27916E-02
53	0.0	0.24050E-01	0.0	-0.64398E-02
54	0.0	0.0	0.0	-0.28044E-01

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